

Summer Block 3

Decimals

Small steps

Step 1

Use known facts to add and subtract decimals within 1

Step 2

Complements to 1

Step 3

Add and subtract decimals across 1

Step 4

Add decimals with the same number of decimal places

Step 5

Subtract decimals with the same number of decimal places

Step 6

Add decimals with different numbers of decimal places

Step 7

Subtract decimals with different numbers of decimal places

Step 8

Efficient strategies for adding and subtracting decimals

Small steps

Step 9

Decimal sequences

Step 10

Multiply by 10, 100 and 1,000

Step 11

Divide by 10, 100 and 1,000

Step 12

Multiply and divide decimals – missing values

Use known facts to add and subtract decimals within 1

Notes and guidance

In this small step, children add and subtract decimals within 1 whole using known facts. They will move on to using a formal method to add and subtract decimals later in this block.

Through unitising, children are able to make connections between whole numbers and decimals. For example, 7 ones + 9 ones = 16 ones, therefore 7 hundredths + 9 hundredths = 16 hundredths. Ensure that children have a good understanding of place value, as a common error is to ignore the place value of decimals, leading to incorrect calculations such as $0.48 + 0.3 = 0.51$. Using a stem sentence allows children to recognise that the unit they are adding or subtracting must be the same, so in this example 48 hundredths + 30 hundredths = 78 hundredths. Hundred squares and place value charts are useful representations to support children when adding and subtracting decimals within 1 whole.

Things to look out for

- Children may add digits together irrespective of which place value column they are in, e.g. $0.45 + 0.3 = 0.48$
- Children may rely on using formal written methods to add/subtract decimals within 1 instead of using known facts.

Key questions

- How can you use the hundred square to help you with the addition/subtraction?
- What whole number calculation can you compare this calculation to?
- How can you convert between tenths and hundredths?
- Which known facts can help you with this calculation?
- What is 1 hundredth more than your number?
- What is 2 tenths less than your number?

Possible sentence stems

- _____ tenths = _____ hundredths
- _____ ones + _____ ones = _____ ones,
so _____ tenths + _____ tenths = _____ tenths
- _____ hundredths – _____ hundredths = _____ hundredths

National Curriculum links

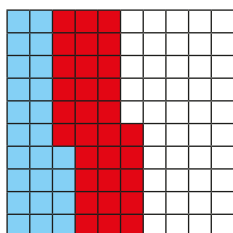
- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Solve problems involving number up to 3 decimal places

Use known facts to add and subtract decimals within 1

Key learning

- Complete the sentences.

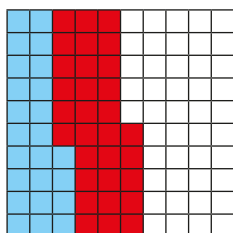
- Each square in this hundred square represents 1 whole.



_____ ones + _____ ones = _____ ones

_____ + _____ = _____

- Each square in this hundred square represents one-hundredth of the whole.



_____ hundredths + _____ hundredths =

_____ hundredths

_____ + _____ = _____

What is the same and what is different about the hundred squares?

- Use a hundred square to work out the calculations.

$$0.72 + 0.13$$

$$0.16 + 0.08$$

$$0.28 + 0.49$$

$$0.62 + 0.19$$

- Here is a number.

Ones	Tenths	Hundredths
	0.1 0.1 0.1	0.01 0.01 0.01
	0.1 0.1	0.01 0.01 0.01

- What is 3 tenths less than this number?
- What is 0.02 more than this number?

- Max uses known facts to complete the subtraction.

$$86 - 24 = 62, \text{ so } 0.86 - 0.24 = 0.62$$

Use known facts to work out the calculations.

- 0.89 – 0.41
- £0.45 – £0.27
- 37 hundredths more than 14 hundredths
- 72 hundredths – 19 hundredths

- Mo and Dora are working out $0.76 - 0.3$

Mo $0.76 - 0.3 = 0.73$

Dora $0.76 - 0.3 = 0.46$

Who is correct?

How do you know?

Use known facts to add and subtract decimals within 1

Reasoning and problem solving

Is the statement true or false?

$$\begin{aligned} 21 \text{ hundredths} - 10 \text{ hundredths} \\ = 0.21 - 0.1 \end{aligned}$$

Explain your answer.

True

Whitney is working out $0.4 - 0.07$

I cannot
work out $0.4 - 0.07$,
because 7 is
greater than 4



No

Do you agree with Whitney?

Explain your answer.

Tiny is working out $0.57 + 0.4$



$$\begin{aligned} 57 + 4 &= 61 \\ \text{So } 0.57 + 0.4 &= 0.61 \end{aligned}$$

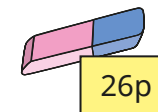
0.97

What mistake has Tiny made?

What is the correct answer?

Brett has £0.89

He buys a pencil and a rubber.



£0.14

How much money does Brett have left?

Give your answer in pounds and pence.

Complements to 1

Notes and guidance

In this small step, children find complements to 1 for numbers with up to 3 decimal places.

It is important for children to see the links with number bonds to 10, 100 and 1,000, and it may be useful to revise these first. The use of ten frames and hundred squares can support children to see the number bonds to 10 and 100 and the corresponding number bonds to 1 for numbers with 1 or 2 decimal places respectively. The number bonds to 1,000 and corresponding 3-decimal place bonds to 1 can be more challenging, but children should be encouraged to apply the same principles as for numbers with fewer decimal places.

Things to look out for

- When finding a complement to 1, children may assume that they need to find the bond to 10 in each place value column, for example $0.365 + 0.745 = 1$
- Children may try to use a formal written method to find complements to 1 instead of using known number bonds.

Key questions

- What number bonds can you use to help you?
- What is the missing number in $64 + \underline{\quad} = 100$?
How does this help you to work out the missing number in $0.64 + \underline{\quad} = 1$?
- What do you need to add to $\underline{\quad}$ to make 10/100/1,000?
So what do you need to add to $\underline{\quad}$ to make 1?
- What is the same and what is different about finding complements to 10/100/1,000 and complements to 1?

Possible sentence stems

- $1 = \underline{\quad}$ tenths = $\underline{\quad}$ hundredths = $\underline{\quad}$ thousandths
- $\underline{\quad}$ ones + $\underline{\quad}$ ones = 10 ones,
so $\underline{\quad}$ tenths + $\underline{\quad}$ tenths = 10 tenths = 1
- $\underline{\quad}$ hundredths/thousandths + $\underline{\quad}$
hundredths/thousandths = 1

National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Solve problems involving number up to 3 decimal places

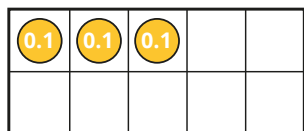
Complements to 1

Key learning

- Each square in the ten frame represents 1 tenth.

The ten frame represents 1 whole.

Complete the statements.



3 tenths + _____ tenths = 10 tenths

10 tenths = 1 whole

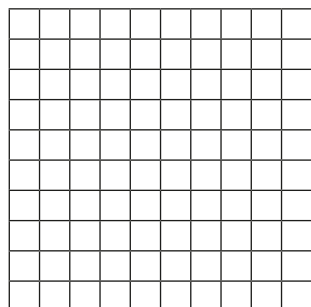
____. ____ + ____ . ____ = 1

Use a ten frame to complete the calculations.

- ▶ $0.8 + \underline{\hspace{1cm}} = 1$
- ▶ $1 = \underline{\hspace{1cm}} + 0.4$
- ▶ $0.1 + \underline{\hspace{1cm}} = 1$
- ▶ $1 = 0.5 + \underline{\hspace{1cm}}$

- Each square in the hundred square represents 1 hundredth of the whole.

Use the hundred square to complete the calculations.



- ▶ $0.55 + \underline{\hspace{1cm}} = 1$
- ▶ $1 = 0.32 + \underline{\hspace{1cm}}$
- ▶ $0.11 + 0.5 + \underline{\hspace{1cm}} = 1$

- Jack is working out $0.763 + \underline{\hspace{1cm}} = 1$

763 ones + 237 ones = 1,000 ones,
so 763 thousandths + 237 thousandths = 1,000 thousandths.
 $0.763 + 0.237 = 1$

Use Jack's method to complete the calculations.

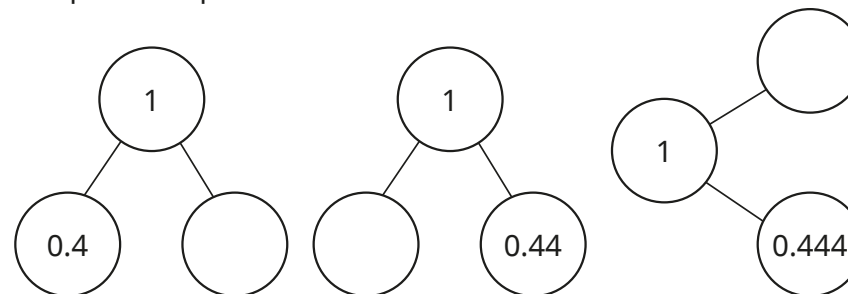
- ▶ $0.356 + \underline{\hspace{1cm}} = 1$
- ▶ $1 = 0.873 + \underline{\hspace{1cm}}$
- ▶ $\underline{\hspace{1cm}} + 0.456 = 1$
- ▶ $1 = \underline{\hspace{1cm}} + 0.048$

- Complete the calculations.

- ▶ $0.3 + \underline{\hspace{1cm}} = 1$
- ▶ $0.35 + \underline{\hspace{1cm}} = 1$
- ▶ $0.399 + \underline{\hspace{1cm}} = 1$

What is the same and what is different?

- Complete the part-whole models.



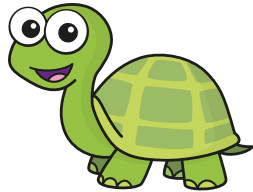
Complements to 1

Reasoning and problem solving

Tiny is working out the missing number.

$$0.333 + \boxed{} = 1$$

The answer is 0.777, because the bond to 10 for 3 is 7



Explain why Tiny is incorrect.
What is the missing number?

0.667

Max, Mo and Annie are baking.

They have 1 kg of flour between them.



Max

I need one quarter of a kilogram of flour.

I need 0.2 kg of flour.



Mo

0.325 kg



Annie

I need 225 g of flour.

What is the mass of the flour that will be left over?

Give your answer in kilograms.

Compare methods with a partner.

Add and subtract decimals across 1

Notes and guidance

In this small step, children add and subtract decimals that cross 1

For some numbers, using known facts is again a useful strategy, for example $6 + 7 = 13$, so $0.6 + 0.7 = 1.3$. Children can also use their experience from the previous step of finding complements to 1, using the “make 1” strategy to help them add and subtract. This requires a secure understanding of flexible partitioning, which allows them to partition decimals into appropriate numbers. For example, when calculating $0.64 + 0.45$, children can use their knowledge of finding complements to 1: $0.64 + 0.36 = 1$, therefore 0.45 should be partitioned into 0.36 and 0.09, leading to $0.64 + 0.36 = 1$ and $1 + 0.09 = 1.09$. Part-whole models or other diagrams can be used to support this. Similarly, when subtracting decimals, encourage children to subtract to get to 1 first, then subtract the remaining decimal.

Things to look out for

- Children may make place value errors, for example using $6 + 7 = 13$ to deduce $0.6 + 0.7 = 0.13$
- Children may make errors with complements to 1 by looking at columns individually, for example thinking that adding 0.38 to 0.72 makes 1

Key questions

- How could partitioning one of the numbers help you?
- How do you decide which number to partition?
- How could you partition this number to help find a complement to 1? What number is left?
- How can you use your number bond knowledge to help you?
- What is the same and what is different about crossing 1 when adding and subtracting decimals?

Possible sentence stems

- _____ can be partitioned into _____ and _____
- The first number is _____ away from 1
The second number can be partitioned into _____ and _____
The total is $1 + \text{_____} = \text{_____}$
- I can subtract _____ to get to 1 and then subtract _____ from 1

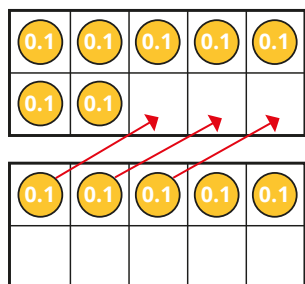
National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Solve problems involving number up to 3 decimal places

Add and subtract decimals across 1

Key learning

- Huan is using ten frames to work out $0.7 + 0.5$



$$\begin{aligned} 0.7 + 0.3 &= 1 \\ 1 + 0.2 &= 1.2 \\ 0.7 + 0.5 &= 1.2 \end{aligned}$$

Use Huan's method to work out the additions.

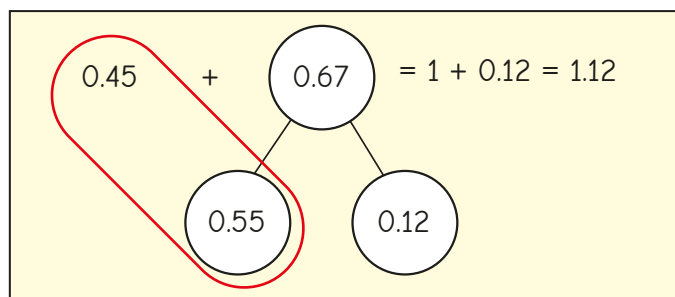
$0.8 + 0.6$

$0.2 + 0.9$

$0.4 + 0.9$

$0.8 + 0.9$

- Dani is finding a complement to 1 to work out $0.45 + 0.67$



Use Dani's method to work out the additions.

$0.74 + 0.78$

$0.74 + 0.42$

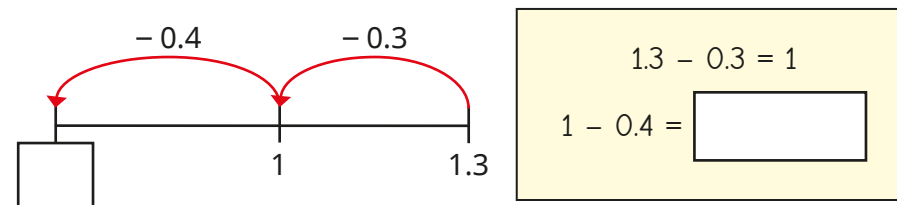
$0.57 + 0.65$

$0.81 + 0.46$

- Scott is using a number line to subtract decimals crossing 1

He is working out $1.3 - 0.7$

Complete Scott's workings.



Use Scott's method to work out the subtractions.

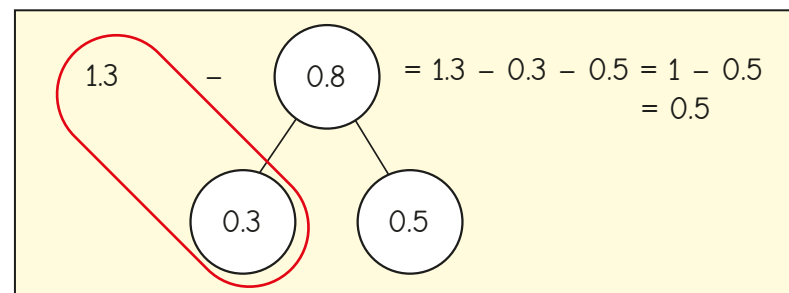
$1.3 - 0.4$

$1.5 - 0.9$

$1.6 - 0.8$

$1.2 - 0.5$

- Kim uses partitioning to work out $1.3 - 0.8$



Use Kim's method to work out the subtractions.

$1.1 - 0.4$

$1.24 - 0.59$

$1.36 - 0.48$

Add and subtract decimals across 1

Reasoning and problem solving

Max is working out $0.78 + 0.43$



I will add
4 tenths to 7 tenths,
3 hundredths to
8 hundredths and then
add my answers.

Will Max's method work?

How do you know?

Yes

Tiny is working out $0.8 + 0.4$



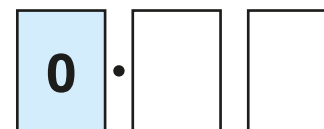
$8 + 4 = 12$, so
 $0.8 + 0.4 = 0.12$

What mistake has Tiny made?

What is the correct answer?

1.2

You need a partner and a 6-sided
dice for this game.



Take turns to roll the dice twice and
create a decimal number less than 1
using the digits you rolled.

Repeat to create a second number.

Add your two numbers together.

Repeat until you have each added four
numbers.

The winner is the person whose total is
the closest to 1.5 **without** going
above 1.5

Compare
strategies as
a class.

Add decimals with the same number of decimal places

Notes and guidance

In this small step, children add decimal numbers with the same number of decimal places, using the formal written method for the first time.

Children begin by looking at calculations with no exchanges before moving on to calculations that involve exchanges and numbers with up to 3 decimal places. Place value charts and counters are extremely helpful in ensuring that children understand the value of each digit and when an exchange is needed. When there are 10 or more in a place value column, children can physically exchange, for example, 10 tenths for 1 whole. They could also compare using column methods for integers and decimals, for example comparing $46 + 38$ with $4.6 + 3.8$.

Children also perform decimal calculations with money, converting amounts in pence to pounds if necessary.

Things to look out for

- Children may not line up the columns correctly, particularly if the calculation involves zero as a placeholder.
- Children may position the decimal point incorrectly.
- Children may forget to add the exchanged digit.

Key questions

- How can you represent this calculation using a place value chart?
- What happens when there are 10 or more counters in a place value column? What is the same and what is different in the formal written method?
- Why is the position of the decimal point important?
- Why is it important to line up the columns?
- Will this addition involve an exchange? How do you know?

Possible sentence stems

- _____ ones + _____ ones = ones,
so _____ tenths + _____ tenths = _____ tenths
- The greatest number I can have in any column is _____
If the total is greater than _____, I need to make an _____

National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Solve problems involving number up to 3 decimal places

Add decimals with the same number of decimal places

Key learning

- Use the place value chart and the column method to work out $3.42 + 4.14$

Ones	Tenths	Hundredths
1 1 1 1	0.1 0.1 0.1 0.1	0.01 0.01
1 1 1 1	0.1	0.01 0.01 0.01 0.01

	3	4	2	
+	4	1	4	

Use place value charts and the column method to work out the additions.

$5.2 + 3.6$

$4.13 + 2.45$

$3.146 + 1.513$

$4.054 + 3.624$

- Use the place value chart and the column method to add 2.83 and 4.41

Ones	Tenths	Hundredths
1 1 1 1	0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1	0.01 0.01 0.01
1 1 1 1	0.1 0.1 0.1 0.1	0.01

	2	8	3	
+	4	4	1	

Use place value charts and the column method to work out the additions.

$4.7 + 3.6$

$3.29 + 4.65$

$8.714 + 2.613$

$15.86 + 13.48$

- Use the column method to work out the additions.

	4	4	2	
+	3	5	3	

	4	5	5	
+	3	0	7	

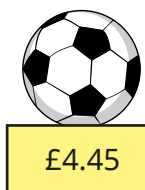
	4	6	0	2
+	3	9	4	9

- Filip buys a hat and a scarf.



How much does it cost him altogether?

- Aisha buys three of these items.



What is the most she could pay?

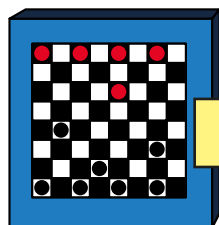
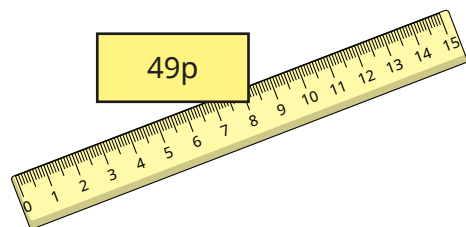
What is the least she could pay?

Add decimals with the same number of decimal places

Reasoning and problem solving



Tiny is working out the total cost of a ruler and a game.



		1	1	3	5
	+	4	9	.	
		6	0	3	5
		1			

Explain Tiny's mistake.

Tiny has put the price of the ruler in the wrong columns.

$$49\text{p} = £0.49$$

Use the digit cards to complete the column addition.

You may use each digit only once in each addition.

0	1	2	3	4
5	6	7	8	9

			.		
	+		.		
			.		

What is the greatest possible sum?

What is the smallest possible sum?

Is there more than one way of creating each total?

greatest:
18.39

smallest:
1.59

Subtract decimals with the same number of decimal places

Notes and guidance

In this small step, children subtract numbers with the same number of decimal places, using the formal written method for the first time.

As with addition, children first look at calculations with no exchanges, before moving on to calculations that involve exchanges and numbers up to 3 decimal places. Place value charts and counters continue to support understanding of the value of each digit and when an exchange is needed. Again, children should look at the formal and practical methods alongside each other to begin with. When an exchange is needed, children can physically exchange, for example, 1 one for 10 tenths. They could also compare using column methods for integers and decimals, for example comparing $76 - 28$ with $7.6 - 2.8$

Give children opportunities to apply subtraction to real-life contexts, for example using measures and money.

Things to look out for

- Children may not line up the columns correctly, particularly when zero is used as a placeholder.
- When subtracting using the column method, children may just find the difference between the digits, rather than making an exchange when necessary, for example $4.5 - 3.8 = 1.3$

Key questions

- What are _____ ones/tenths/hundredths subtract _____ ones/tenths/hundredths?
- Will you need to make an exchange in this subtraction?
How do you know?
- What can you exchange 1 one/tenth/hundredth for?
- Why is the position of the decimal point important?
- What does zero in a place value column mean? How does this affect a subtraction?

Possible sentence stems

- _____ ones/tenths subtract _____ ones/tenths is equal to _____ ones/tenths.
- I need to make an exchange because ...
- I need to exchange 1 _____ for 10 _____

National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Solve problems involving number up to 3 decimal places

Subtract decimals with the same number of decimal places

Key learning

- Use the place value chart and the column method to work out $4.23 - 2.12$

Ones	Tenths	Hundredths
1 1 1 1	0.1 0.1	0.01 0.01 0.01

	4	2	3	
	-	2	1	2

Did you need to make any exchanges?

- Use the place value chart and the column method to work out $6.35 - 4.83$

Will you need to make any exchanges?

Ones	Tenths	Hundredths
1 1 1 1 1 1	0.1 0.1 0.1	0.01 0.01 0.01 0.01 0.01

	6	3	5	
	-	4	8	3

Use a place value chart and a column method to work out the subtractions.

$$5.7 - 2.4$$

$$8.56 - 3.37$$

$$8.313 - 2.614$$

$$13.24 - 12.06$$

- Use the column method to work out the subtractions.

	6	4		
	-	3	8	

	7	3	0	4
	-	3	9	1

	5	0	5	
	-	2	1	5

	2	4	6	3
	-	1	7	8

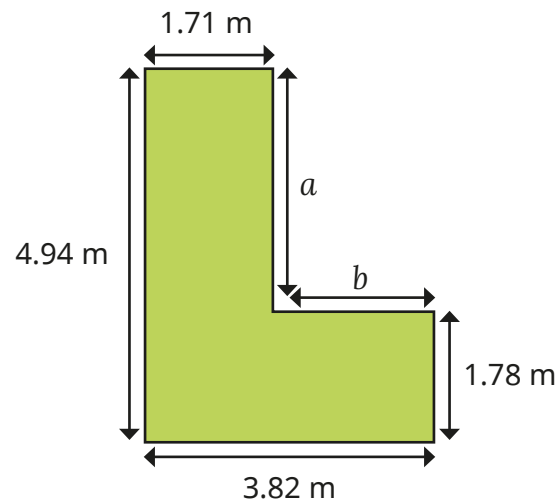
- Tom has £12.45
He buys a football for £6.99
How much money does he have left?
Compare methods with a partner.
- Annie and Amir are doing a sponsored bike ride.
Annie cycles 8.47 miles.
Amir cycles 5.95 miles.
How much further does Annie cycle than Amir?



Subtract decimals with the same number of decimal places

Reasoning and problem solving

Work out the lengths of sides a and b .



What is the perimeter of the hexagon?

$$a = 3.16 \text{ m}$$

$$b = 2.11 \text{ m}$$

$$17.52 \text{ m}$$

Dexter and Nijah have some money.

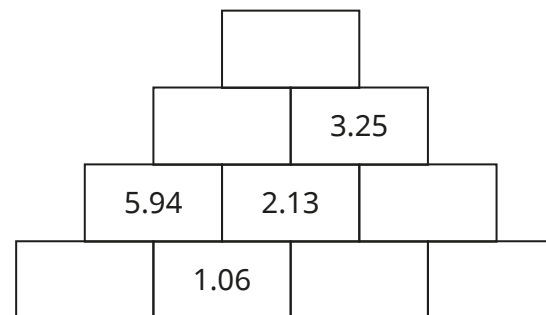
Dexter has £3.45 more than Nijah.

They have £12.45 altogether.

How much money does Nijah have?

£4.50

In the number pyramid, each number is the sum of the two numbers below it.



Complete the number pyramid.

11.32

8.07

1.12

4.88 1.07 0.05

Add decimals with different numbers of decimal places

Notes and guidance

In this small step, children extend their knowledge of adding decimal numbers to include numbers with a different number of decimal places.

Emphasise the importance of lining up the decimal point in order to ensure that digits with the same place value are also aligned. A place value chart is a useful representation to reinforce this, as children can see the value of each digit in the correct place value column. Children could be encouraged to “fill” empty columns with trailing zeros to promote an understanding of using the zero as a placeholder and making it easier to see how the numbers line up.

Children could also use estimation to think about whether their answers are sensible.

As in previous steps, it may be useful to begin with examples that do not require an exchange, so that children can focus on the new learning of adding numbers with a different number of decimal places.

Things to look out for

- Children may not line up digits correctly.
- Children may put trailing zeros in the wrong place, for example writing 8.6 as 8.06 instead of 8.60

Key questions

- How can you show this addition on a place value chart?
- What happens when there are 10 or more counters in a place value column?
- Why is the position of the decimal point important?
- Why is it important to line up the columns?
- Will this addition involve an exchange? How do you know?
- What could you add to the spaces that do not contain a digit, to help you?

Possible sentence stems

- When adding two decimal numbers, I need to keep the _____ in line.
- _____ tenths + _____ tenths = _____ tenths, so I do/do not need to make an exchange.

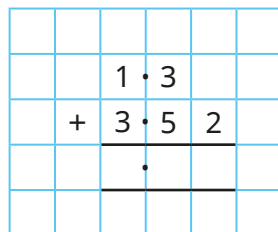
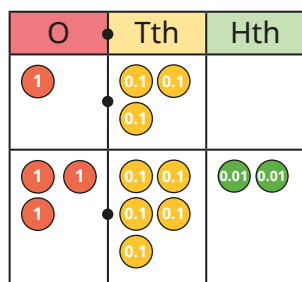
National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Solve problems involving number up to 3 decimal places

Add decimals with different numbers of decimal places

Key learning

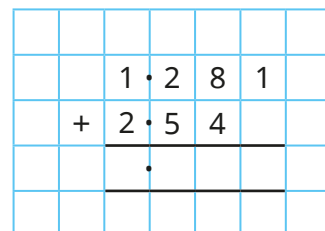
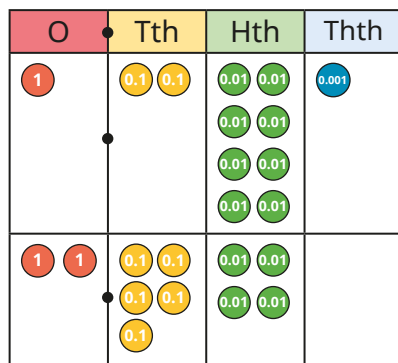
- Use the place value chart and column method to work out $1.3 + 3.52$



Work out the additions.

- $5.7 + 3.16$ ► $2.017 + 3.5$ ► $4.61 + 3.372$

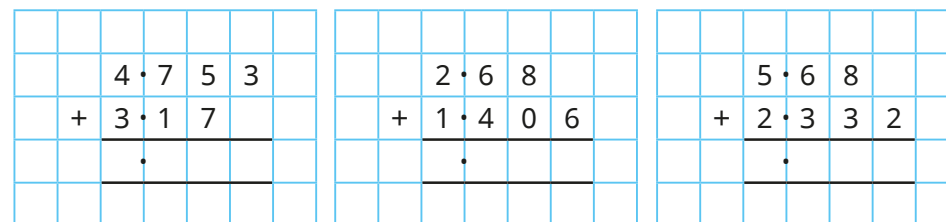
- Use the place value chart and column method to work out $1.281 + 2.54$



Work out the additions.

- $4.7 + 3.56$ ► $2.8 + 1.317$ ► $3.595 + 4.62$

- Use the column method to work out the additions.



- Complete the bar model.



- Sam is cycling in a race.
She has cycled 3.145 km and has 4.1 km left to cycle.
What is the total distance of the race?

- Work out the additions.

$$8.7 \text{ m} + 5.29 \text{ m}$$

$$0.63 \text{ litres} + 0.8 \text{ litres}$$

$$6.3 \text{ kg} + 2.75 \text{ kg}$$

$$5.173 \text{ km} + 4.08 \text{ km}$$

Add decimals with different numbers of decimal places

Reasoning and problem solving



Tiny is working out
 $4.144 + 1.4$

What mistake has Tiny made?

What is the correct answer?

		4	.	1	4	4	
	+				1	.	4
		4	.	1	5	8	



5.544

Find a solution to the addition with:

- no exchanges
- 1 exchange
- 2 exchanges

$$\boxed{}.\boxed{}\boxed{} + \boxed{}.\boxed{}\boxed{}\boxed{} = 3.678$$

multiple possible answers, e.g.

1.15 + 2.528 2.28 + 1.398 2.79 + 0.888

Write the additions in the correct columns in the table.

$$9.99 + 0.1$$

$$9.99 + 1$$

$$9.99 + 0.001$$

$$9.99 + 0.01$$

No exchange	Exchange in ones column	Exchange in tenths column	Exchange in hundredths column	Exchange in thousandths column

Some additions may go in more than one column.

Add two more additions to each column, where the numbers have a different number of decimal places.

no exchange: $9.99 + 0.001$

ones column: $9.99 + 1$,
 $9.99 + 0.1$, $9.99 + 0.01$

tenths column:

$9.99 + 0.1$, $9.99 + 0.01$

hundredths column:

$9.99 + 0.01$

Subtract decimals with different numbers of decimal places

Notes and guidance

In this small step, children extend their knowledge of subtracting decimal numbers to include numbers with a different number of decimal places.

It is important that children continue to practise lining up the decimal point carefully and ensure that each digit is in the correct column. A place value chart could be used to reinforce this. In the column method, show children how to “fill” empty columns with zeros, which will support them when exchanges are required. They need to be secure with the fact that, for example, 6 and 6.0 have the same numerical value, as do 4.7 and 4.70 and so on.

Children need a good understanding of column subtraction from previous steps, knowing when to make an exchange – particularly when zeros are involved.

Things to look out for

- Children may not line up digits correctly.
- In calculations such as $7.6 - 2.38$, children may subtract where there are pairs of numbers but just write the last digit, giving the answer of 5.38, instead of writing $7.60 - 2.38$ and making an exchange.
- Children may struggle when multiple exchanges are required, for example $13 - 2.532$

Key questions

- How should the digits be lined up in a column subtraction?
- How do you show that there is nothing in a place value column?
- Do you need to make an exchange? How do you know?
- How do you make an exchange if there is a zero in the column that you want to make the exchange from?
- Is the column subtraction method the most efficient method to use in this example?

Possible sentence stems

- When subtracting two decimal numbers, I need to keep the _____ in line.
- If I need to subtract hundredths and there is no digit in the hundredths column, I can put in a _____ and then make an _____

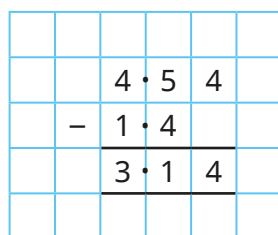
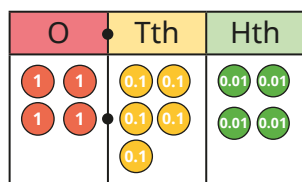
National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Solve problems involving number up to 3 decimal places

Subtract decimals with different numbers of decimal places

Key learning

- Alex is using a place value chart and column subtraction to work out $4.54 - 1.4$



Use place value charts and the column method to work out the subtractions.

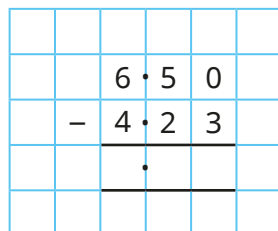
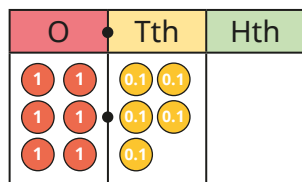
$$2.65 - 1.4$$

$$5.42 - 1.3$$

$$7.326 - 4.01$$

$$8.72 - 4.1$$

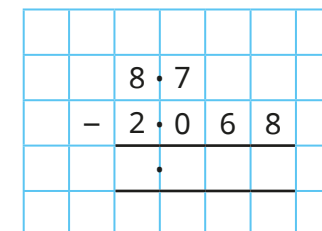
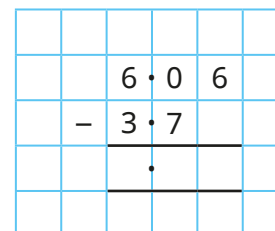
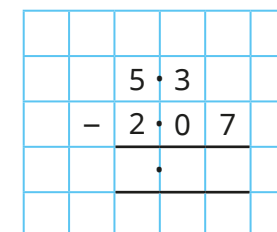
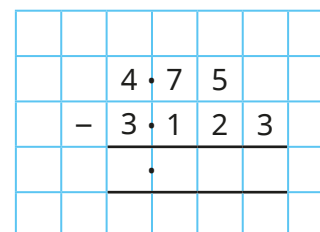
- Teddy is using a place value chart and column subtraction to subtract 4.23 from 6.5



Why can Teddy write 6.5 as 6.50?

Complete the calculation using place value counters to help you.

- Use the column method to work out the subtractions.



- Eva buys a bag of apples costing £4.27

She pays with a £10 note.

How much change does she get?

- Work out the subtractions.

$$5,000 \text{ g} - 3,200 \text{ g} = \text{_____ g} \quad 5 \text{ kg} - 3.2 \text{ kg} = \text{_____ kg}$$

$$450 \text{ cm} - 255 \text{ cm} = \text{_____ cm} \quad 4.5 \text{ m} - 2.55 \text{ m} = \text{_____ m}$$

$$550 \text{ ml} - 60 \text{ ml} = \text{_____ ml} \quad 0.55 \text{ l} - 0.06 \text{ l} = \text{_____ l}$$

Subtract decimals with different numbers of decimal places

Reasoning and problem solving

Tiny is working out $4.9 - 3.84$



		4	.	9	
	-	3	.	8	4
		1	.	1	4

What mistake has Tiny made?
Work out the correct answer.

1.06

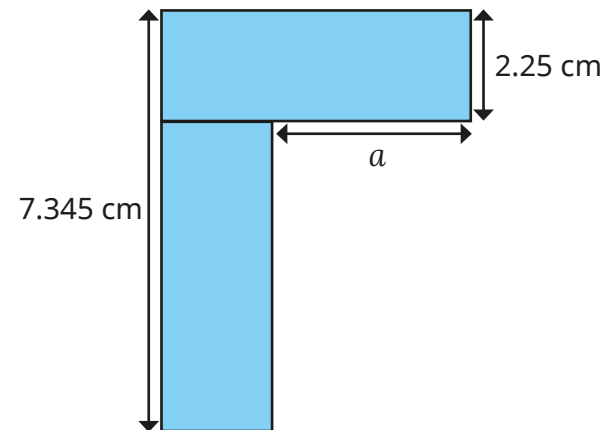
Rosie, Mo and Dora each have some money.

- Rosie has £1.63 more than Mo.
- Dora has £4 more than Rosie.
- Dora has £7.60

How much money do they have altogether?

£13.17

The shape is made of two identical rectangles.



2.845 cm

Find the length of the part marked a .

Efficient strategies for adding and subtracting decimals

Notes and guidance

In this small step, children explore a range of different calculation strategies to solve addition and subtraction problems, making decisions about which strategy would be the most effective for each problem.

Encourage children to consider the question carefully rather than automatically choosing the same option every time. They can experiment by solving the same calculation in a number of ways and considering which way was the most efficient and why. In particular, discuss when mental strategies are more appropriate than written, for example when compensation can be used, such that adding 9.99 can be simplified to add 10 and then subtract 0.01. Number lines are useful to support this approach.

Things to look out for

- Children may automatically use formal written methods, even when they are less efficient.
- Children may not transfer strategies used with integers to decimals without explicit teaching.
- When working mentally, children may make place value errors.

Key questions

- Do you need to make an exchange?
- What methods could you use?
Which is most efficient for this calculation?
- When would you use a mental method?
- When would you use an informal jotting such as a number line?
- When would a formal method be more efficient?
- What integer is 9.9 close to?
How can this help with the calculation?
- How could partitioning help with this calculation?

Possible sentence stems

- _____ is close to _____, so I can change the calculation to _____
- I will work this out using _____ because ...

National Curriculum links

- Solve problems involving number up to 3 decimal places

Efficient strategies for adding and subtracting decimals

Key learning

- Dani uses a place value chart and a written method to work out $43 + 1.45$

T	O	Tth	Hth
10 10 10 10	1 1 1		
	1	0.1 0.1 0.1 0.1	0.01 0.01 0.01 0.01 0.01

		4	3	.			
	+		1	.	4	5	
		4	4	.	4	5	

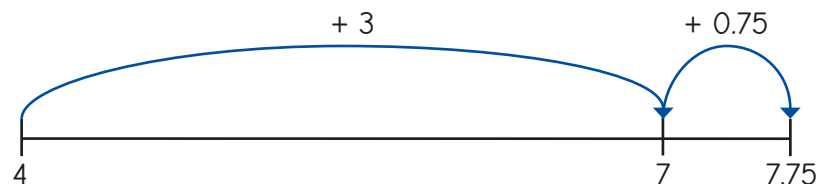
Could Dani have worked the answer out using a mental method?

Which of these calculations could you work out mentally?

For which calculations would you use a written method?

- ▶ $8.2 + 3.1$ ▶ $6.9 + 0.45$ ▶ $9.8 - 4$ ▶ $90.8 - 0.45$
- ▶ $18.02 + 34.19$ ▶ $6.7 + 0.25$ ▶ $9.8 - 4.56$ ▶ $9.8 - 0.4$

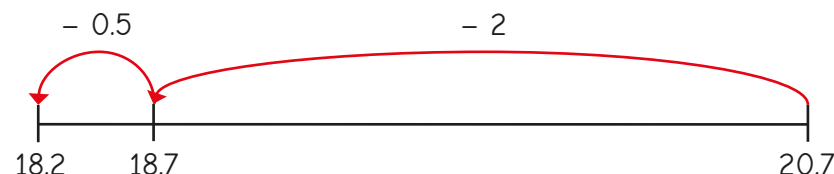
- Whitney uses a number line to work out $4 + 3.75$



Use Whitney's method to work out the additions.

- ▶ $7 + 0.65$ ▶ $4 + 3.2$ ▶ $12 + 4.63$ ▶ $19 + 8.784$

- Brett is counting back along a number line to work out $20.7 - 2.5$



Use Brett's method to work out the subtractions.

- ▶ $16.8 - 2.5$ ▶ $12.9 - 4.3$ ▶ $14.6 - 8.05$ ▶ $15.75 - 8.32$

- Work out $8.4 + 3.42$ using:

- a mental method
- a number line
- the column method.

Which method do you think is best?

Would this be the best method to work out $8.4 - 3.42$?

Explain your answer.

- Use your preferred method to work out the calculations.

$$43 - 2.14 + 0.86$$

$$23 + 4.105$$

$$19 - 0.25$$

$$19 - 17.37$$

Compare methods with a partner.

Efficient strategies for adding and subtracting decimals

Reasoning and problem solving

For each calculation, decide if you would use a mental method, an informal jotting or the formal written method.

$57.9 + 4.8$

$12.8 + 5.4$

$5.6 + 2.1$

$9.5 - 4.3$

$8.6 - 7.7$

$3.25 - 1.37$

Mental method	Informal jotting	Formal written method

Explain your choices.

Add one more calculation to each column.

Discuss as a class.

Work out the missing digits.

		3	1	.		0	
	-			.	3	7	
		2	9	.	6	3	

$31.00 - 1.37$



Tiny is working out $63.7 - 9.9$

$$\begin{aligned}
 63.7 - 9.9 &= 63.7 - 10 - 0.1 \\
 &= 53.7 - 0.1 \\
 &= 53.6
 \end{aligned}$$

What mistake has Tiny made?

How could you work out the change from £20 when you spend £6.99?

Tiny should have subtracted 10 and then **added** 0.1

Decimal sequences

Notes and guidance

In this small step, children combine their knowledge of number sequences and decimals to explore decimal sequences.

Given a range of sequences, children look for patterns and use and find simple rules that involve adding or subtracting a decimal each time. It is important to note that they are not expected to generate algebraic expressions at this stage. Children should, however, use the language associated with sequences such as “term” and “rule”. They should make predictions about the next term or subsequent terms in a sequence or, given different terms in a sequence, work backwards to find previous terms. Number lines are useful for representing sequences.

This step supports children’s understanding of counting in decimals, particularly across an integer, and prepares them for further study of sequences in Year 6

Things to look out for

- Children may make errors when crossing an integer boundary, for example 0.3, 0.6, 0.9, 0.12
- When looking for terms earlier in a sequence, children may use the operation for the rule instead of the inverse operation, for example adding when they need to subtract.

Key questions

- Are the terms increasing or decreasing in value?
- Are the terms increasing or decreasing by the same amount each time? If so, by how much?
- What will the next term in the sequence be?
- What will the _____ term in the sequence be?
- How can you tell if you need to make an exchange?
- How can you work out the previous term in a sequence? Does it make a difference if the sequence is increasing or decreasing?

Possible sentence stems

- Each term is _____ than the previous term.

The difference between the terms is _____

As the sequence is increasing/decreasing, I need to add/ subtract _____ to work out the next term.

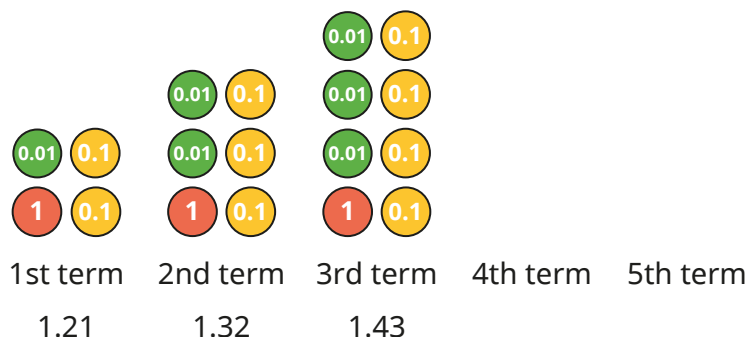
National Curriculum links

- Read, write, order and compare numbers with up to 3 decimal places
- Solve problems involving number up to 3 decimal places

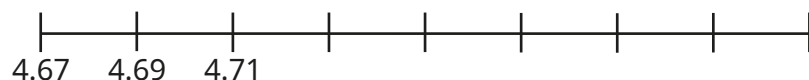
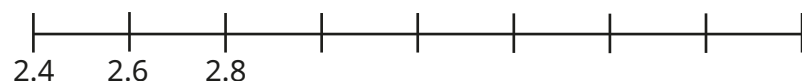
Decimal sequences

Key learning

- Complete the sequence.



- Complete the number lines.



- Write the rule for each sequence.

- ▶ 3.4, 3.6, 3.8, 4
- ▶ 3.4, 3.2, 3, 2.8
- ▶ 3.4, 3.42, 3.44, 3.46
- ▶ 3.4, 3.38, 3.36, 3.34

Work out the next term in each sequence.

- Use the rule to find the missing terms in the sequences.

- ▶ Rule: add 0.3

0.4, _____, _____, _____, _____

- ▶ Rule: add 0.25

_____, _____, 3.75, _____, _____

- ▶ Rule: subtract 1.1

_____, _____, _____, 7.8, _____

- A library charges a £1.50 fine if a book is not returned on the due date, and 15p per day for every day after that.

Use the sequence to work out the fine for a book that is one week overdue.

£1.50, £1.65, _____, _____, _____, _____, _____

- The 1st term of a sequence is 0.7 and the 3rd term is 1

What is the 2nd term of the sequence?

What is the 5th term?

Decimal sequences

Reasoning and problem solving

Here is a sequence.

3.5, 3.7, 3.9 ...



The next term is 3.11

Explain why Kim is wrong.
What is the next term?

4.1

Here is a sequence.

9.48, 9.52, 9.56, 9.6 ...



The number 9.7 will be in this sequence.

Do you agree with Jack?
Explain your answer.

No

0.83

0.78

0.93

0.88

Put the cards in order to make a sequence.

What is the rule?

Could there be a different sequence and a different rule?

0.78, 0.83, 0.88, 0.93
rule: add 0.05

0.93, 0.88, 0.83, 0.78
rule: subtract 0.05

Huan and Alex are writing number sequences starting at zero.

- Huan's rule is + 0.9
- Alex's rule is + 1.2

What is the first number they will both write?

What other numbers will they both write?

3.6

7.2, 10.8 ...
all multiples of 3.6

Multiply by 10, 100 and 1,000

Notes and guidance

In this small step, children learn to multiply decimals by 10, 100 and 1,000

Children multiplied integers by 10 and 100 in Year 4 and moved on to multiply by 1,000 in the Autumn term of Year 5. Despite this experience, they may still make the mistake of over-generalising and simply “adding zeros”. Concrete resources and stem sentences can be used to enable children to make accurate generalisations about what happens to the digits in a number when they multiply by 10, 100 or 1,000. Representations such as place value charts allow children to physically move plain counters to the left and recognise that all digits move, for example, 1 place to the left when multiplying by 10. They can also use a Gattegno chart to recognise that multiplying by 10 and “10 times the size” is the same.

Things to look out for

- Children may assume that they add a zero to the original number when multiplying by 10
- Children may “move the decimal point” instead of recognising that it is the digits that increase in value when multiplying by 10, 100 and 1,000

Key questions

- What is the value of each digit in the number?
- How many places to the left do the counters move when you multiply by 10/100/1,000?
- Where would the digits move to if you multiplied the number by 10/100/1,000?
- How many times greater than _____ is _____?
- If you multiply a number by 10 and then multiply the answer by 10, how many times greater than the original number is your final answer?

Possible sentence stems

- To multiply by 10/100/1,000, I move all the digits _____ places to the left.
- 10 times greater than _____ is _____
- Multiplying by 100/1,000 is the same as multiplying by 10 _____ times.

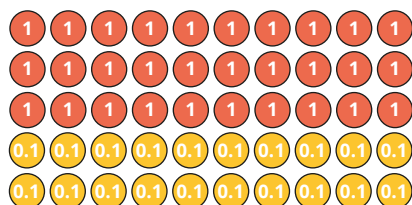
National Curriculum links

- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

Multiply by 10, 100 and 1,000

Key learning

- The place value counters show 3.2 multiplied by 10



► Can you make any exchanges?

► Complete the sentences.

_____ multiplied by 10 is equal to _____

_____ is 10 times the size of _____

- Use the place value chart to multiply 3.24 by 10, 100 and 1,000

T	H	T	O	Tth	Hth
			3	2	4

Complete the sentence.

When you multiply by _____, you move the counters _____ places to the left.

- Use a place value chart to multiply the decimals by 10, 100 and 1,000

4.24	2.401	4.21
------	-------	------

- Mo is using a Gattegno chart to work out 4.9×10

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

$$4 \times 10 = 40$$

$$0.9 \times 10 = 9$$

$$\text{So } 4.9 \times 10 = 49$$

Use the Gattegno chart to work out the multiplications.

- | | | |
|--------------------|--------------------|---------------------|
| ► 0.6×10 | ► 2.4×10 | ► 1.35×10 |
| 0.6×100 | 2.4×100 | 1.35×100 |
| $0.6 \times 1,000$ | $2.4 \times 1,000$ | $1.35 \times 1,000$ |

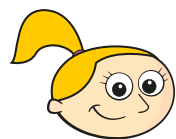
What patterns do you notice?

- Multiply each number by 10, 100 and 1,000

3.14	0.13	0.033
------	------	-------

Multiply by 10, 100 and 1,000

Reasoning and problem solving



Multiplying by
1,000 is the same
as doing
 $\times 10 \times 10 \times 10$

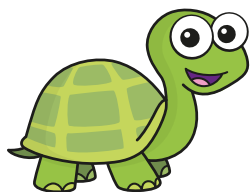
Do you agree with Eva?

Explain your answer.

Yes

Tiny is multiplying by 10

$3.104 \times 10 = 31.4$



What mistake has Tiny made?

What is the correct answer?

31.04

Without calculating, write $<$, $>$ or
 $=$ to make the statements correct.

4.732×100 47.32×10

10×13.82 $1,000 \times 1.382$

$0.723 \times 10 \times 10$ 100×0.723

$1,000 \times 3.81$ 30.81×100

Explain your reasoning.

=
<
=
>

Scott has £4.87

Tom has 10 times as much
money as Scott.

How much more money does
Tom have than Scott?

£43.83

Divide by 10, 100 and 1,000

Notes and guidance

In this small step, children explore dividing integers and decimal numbers by 10, 100 and 1,000. This builds on their learning from Year 4, where they learned to divide 1- and 2-digit numbers by 10. Children should begin to recognise the links with multiplying by 10, 100 and 1,000 and notice the inverse relationship. Concrete resources and stem sentences can be used to enable children to make accurate generalisations about what happens to the digits in a number when they divide by 10, 100 or 1,000. A place value chart allows children to physically move counters to the right and recognise that all of the digits move, for example, 2 places to the right when dividing by 100. Children can also use a Gattegno chart to recognise that dividing by 10 and “one-tenth of the size” is the same.

Things to look out for

- Children may make errors with zero placeholders, for example $30.4 \div 10 = 3.4$
- Children may mix up the rules for multiplication and division.
- Children may “move the decimal point” instead of recognising that it is the digits that decrease in value when dividing by 10, 100 and 1,000

Key questions

- What is the value of each digit in the number?
- If you divide by 10/100/1,000, how many places to the right do the counters move?
- Where would the digits move to if you divided the number by 10/100/1,000?
- How many times smaller is _____ than _____?
- If you divide a number by 10 and then divide the answer by 10, how many times smaller than the original number is your final answer?

Possible sentence stems

- To divide by 10/100/1,000, I move all the digits _____ places to the right.
- _____ is one-tenth the size of _____
- Dividing by 100/1,000 is the same as dividing by 10 _____ times.

National Curriculum links

- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

Divide by 10, 100 and 1,000

Key learning

- Use the place value chart to divide 14 by 10, 100 and 1,000

T	O	Tth	Hth	Thth
●	●●●●			

Complete the sentence.

When you divide by _____, you move the counters _____ places to the right.

- Use a place value chart and counters to divide the numbers by 10, 100 and 1,000

4	15	301
---	----	-----

- Use the place value chart to complete the divisions.

H	T	O	Tth	Hth	Thth
	2	7	●		
			●		
			●		
			●		

$$27 \div 10 = \underline{\hspace{2cm}}$$

$$27 \div 100 = \underline{\hspace{2cm}}$$

$$27 \div 1,000 = \underline{\hspace{2cm}}$$

- Filip is using a Gattegno chart to work out $5.8 \div 10$

100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

$$5 \div 10 = 0.5$$

$$0.8 \div 10 = 0.08$$

$$5.8 \div 10 = 0.58$$

0.58 is one-tenth the size of 5.8

Use the Gattegno chart to work out the divisions.

- | | | |
|-----------------|------------------|------------------|
| ▶ $42 \div 10$ | ▶ $713 \div 10$ | ▶ $102 \div 10$ |
| $42 \div 100$ | $713 \div 100$ | $102 \div 100$ |
| $42 \div 1,000$ | $713 \div 1,000$ | $102 \div 1,000$ |

What patterns do you notice?

- There are 100 pence in £1

Use this fact to convert the amounts from pence to pounds.

- ▶ 210p = £ _____ ▶ 132p = £ _____ ▶ 2,456p = £ _____

Divide by 10, 100 and 1,000

Reasoning and problem solving

Amir is working out $4.08 \div 10$



The answer is 0.48

0.408

What mistake has Amir made?

What is the correct answer?



Mo divides 72 by 1,000

He then multiplies the answer by 10



I can get to the same answer in one step.

Mo can divide by 100 to get the same answer.

Explain Mo's method.



Here are three rectangles.



The sides of rectangle B are 10 times greater than rectangle A.

The sides of rectangle C are one-hundredth the size of rectangle B.

Work out the side lengths of rectangles B and C.



The perimeter of rectangle A is 1,000 times greater than the perimeter of rectangle C.

Do you agree with Rosie?

Explain your answer.



B: 14 m and 9 m

C: 0.14 m and 0.09 m

No

Multiply and divide decimals – missing values

Notes and guidance

In this small step, children apply their knowledge of multiplying and dividing by 10, 100 and 1,000 to work out missing values. Through the use of concrete resources and stem sentences in the two previous steps, children have generalised what happens to the digits in a number when they multiply and divide by 10, 100 or 1,000. They now use these generalisations to support them to find missing values in calculations. Gattegno charts can be used to recognise how many rows a counter has moved up or down, allowing children to work out if the number is 10, 100 or 1,000 times greater or smaller. A place value chart allows them to physically move counters to the left or right to work out if the number is 10, 100 or 1,000 times greater or smaller. Children should recognise the inverse relationship between multiplying and dividing by 10, 100 and 1,000 and use this to find the missing values.

Things to look out for

- Children may mix up multiplication and division and move counters or digits in the wrong direction.
- Children may make errors with numbers that include zero as a placeholder, especially within numbers such as 3.04

Key questions

- What is the value of each digit?
- How many times smaller is _____ than _____?
- How many times greater is _____ than _____?
- How have the values of the digits changed?
- Has the number been multiplied or divided?
How do you know?
- In which direction have the digits moved? How many places have the digits moved? What does this tell you?

Possible sentence stems

- The digits have moved _____ places to the left/right, so the number has been _____ by _____
- The digits have moved _____ places to the left/right, so the number is _____ times greater/smaller.

National Curriculum links

- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

Multiply and divide decimals – missing values

Key learning

- Use the place value chart to work out the missing value.

$$4.23 \times \underline{\hspace{2cm}} = 42.3$$

T	O	Tth	Hth
	4	2	3

- Use a place value chart and counters to work out the missing values.

$$\begin{aligned}
 &\triangleright 3.45 \times \underline{\hspace{2cm}} = 34.5 & \triangleright 84 \div \underline{\hspace{2cm}} = 0.84 \\
 &\triangleright 4.56 \div \underline{\hspace{2cm}} = 0.456 & \triangleright 1.03 \times \underline{\hspace{2cm}} = 103
 \end{aligned}$$

- Mo divides a number by 100 and ends up with 0.52

H	T	O	Tth	Hth	Thth
		0	5	2	

What number did Mo start with?

- Work out the missing numbers.

$$\begin{aligned}
 &\triangleright \underline{\hspace{2cm}} \div 10 = 4.9 & \triangleright \underline{\hspace{2cm}} \times 10 = 0.45 \\
 &\triangleright 1,000 \times \underline{\hspace{2cm}} = 273 & \triangleright \underline{\hspace{2cm}} \div 100 = 2.103
 \end{aligned}$$

- Dexter uses a Gattegno chart to work out the missing value in the calculation $4.82 \times \underline{\hspace{2cm}} = 482$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

- Complete the sentences.

Each counter moves up _____ rows to get to 482

482 is _____ times the size of 4.82

$$4.82 \times \underline{\hspace{2cm}} = 482$$

- Use the Gattegno chart to work out the missing values.

$$3.4 \times \underline{\hspace{2cm}} = 34 \qquad \underline{\hspace{2cm}} \div 10 = 64.5$$

$$\underline{\hspace{2cm}} \times 5.62 = 5,620 \qquad 4.6 \div \underline{\hspace{2cm}} = 0.046$$

$$1,000 \times \underline{\hspace{2cm}} = 345 \qquad \underline{\hspace{2cm}} \div 100 = 3.02$$

- Complete the calculations.

$$\begin{aligned}
 &\triangleright \underline{\hspace{2cm}} \div 10 = 1.93 \div 100 & \triangleright 34.2 \div \underline{\hspace{2cm}} = 0.342 \times \underline{\hspace{2cm}}
 \end{aligned}$$

Multiply and divide decimals – missing values

Reasoning and problem solving

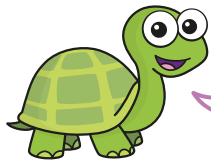
Is the statement true or false?

$$420 \div 100 = 0.042 \times 100$$

Explain your answer.

True

$$\boxed{} \div 100 = 0.594$$



I can multiply 0.594 by 100 to find the missing value.

Yes

Do you agree with Tiny?

Explain your answer.

Ron thinks of a number.

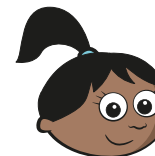


I multiplied my number by 10 and then divided it by 1,000. I ended up with 0.034

What number was Ron thinking of?

3.4

multiplied by 100



I started with the same number as Ron and ended up with 340

What did Sam do to her number?

Summer Block 4

Negative numbers

Small steps

Step 1

Understand negative numbers

Step 2

Count through zero in 1s

Step 3

Count through zero in multiples

Step 4

Compare and order negative numbers

Step 5

Find the difference

Understand negative numbers

Notes and guidance

In this small step, children are introduced to negative numbers for the first time. The focus of this step is exploring negative numbers in real-life contexts, including temperatures, distances above and below sea level and floors in a building that go underground.

In this first step, only vertical representations are used to develop understanding of the concept. Draw attention to the fact that negative numbers can be seen as a reflection of the positive numbers. This will help to avoid the common misconception of counting 3, 2, 1, 0, -10, -9, -8 ...

Careful attention should be paid to language choices and children should be encouraged to say, for example, -3 as “negative three” rather than “minus three”, so that they see negative numbers as numbers rather than operations.

At this stage, children do not need to calculate using negative numbers.

Things to look out for

- As children are often shown scales from positive 10 to negative 10, they may count incorrectly across zero, for example 3, 2, 1, 0, -10, -9, -8 etc.
- Children may only look at the digit and think that, for example, -7 is greater than -2

Key questions

- What are negative numbers? How do you write them?
- As the temperature gets warmer/colder, do the numbers get greater or smaller?
- If zero degrees Celsius is freezing point, how do you write temperatures that are colder than freezing?
- Is -5 colder or warmer than -2? Which temperature is closer to freezing point (zero degrees Celsius)?
- If the ground floor is zero and the first floor is 1, what number represents the basement?
- Which of these floors are above/below the ground floor, -3 and 3?
- If 5 m represents 5 metres above sea level, how do you write 5 metres below sea level?

Possible sentence stems

- Numbers greater than zero are called _____ numbers.
- Numbers less than zero are called _____ numbers.

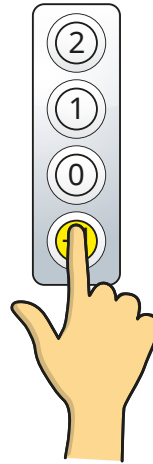
National Curriculum links

- Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

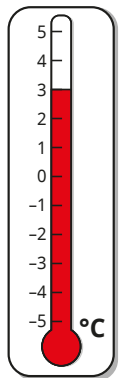
Understand negative numbers

Key learning

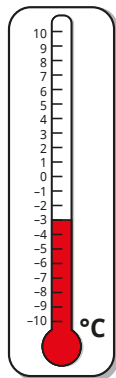
- Mr Rose is in the lift of a building.
He is on the ground floor.
 - What number represents the ground floor?
- Mr Rose wants to go to a shop on the floor above him.
 - What number button does he need to press?
- Mr Rose's car is parked in the car park on the floor below ground level.
 - What number will this be?



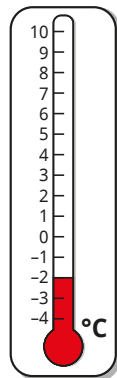
- The thermometers show the temperatures in four cities measured in degrees Celsius ($^{\circ}\text{C}$).



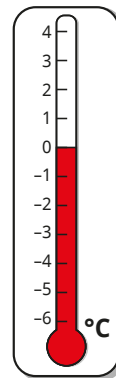
Paris



Oslo



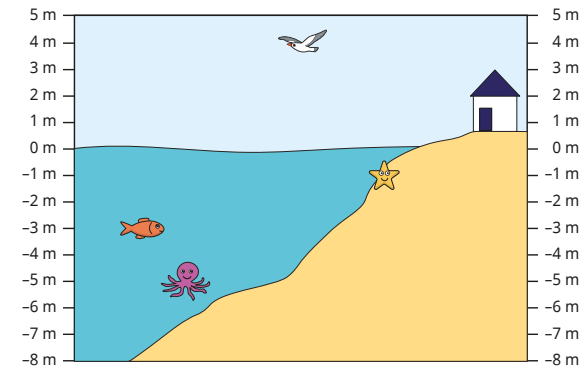
London



Berlin

What temperatures are shown on the thermometers?

- The diagram shows distances above and below sea level.



- At what height is the bird flying?
 - Which creature is at a deeper level, the starfish, fish or octopus?
 - How many metres below the surface of the water is the fish?
- The table shows the temperatures at different times of the day.

Time	Temperature
5 am	-4°C
12 noon	1°C
6 pm	-1°C

Use the clues to work out the temperature at each time.

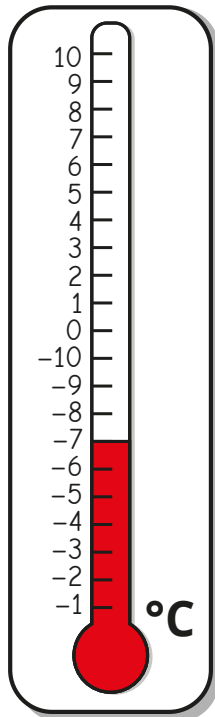
- At 9 am, the temperature was 1 degree warmer than at 5 am.
- At 4 pm, it was colder than at 12 noon but warmer than at 6 pm.
- At 11 pm, it was 1 degree colder than at 5 am.

Understand negative numbers

Reasoning and problem solving



Tiny has labelled the thermometer incorrectly.

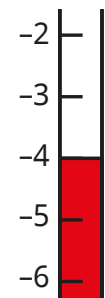


-4 °C

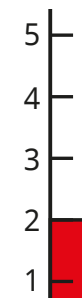
What mistake has Tiny made?
What temperature is shown?



The thermometers show the temperatures in New York and Athens.



New York



Athens

-3 °C, -2 °C, -1 °C,
0 °C or 1 °C

The temperature in Rome is warmer than in New York, but colder than in Athens.

What could the temperature in Rome be?

Count through zero in 1s

Notes and guidance

In this small step, children become more fluent with negative numbers and explore counting both forwards and backwards through zero in 1s. Counting in other multiples through zero will be covered in the next step.

Alongside the vertical representations used in the previous step, children now see horizontal number lines. This will help to reinforce the reflective nature of positive and negative numbers. Use of horizontal number lines provides an opportunity to revisit and develop skills in labelling and identifying numbers on a number line covered in earlier blocks.

Once confident with counting both forwards and backwards through zero on a number line, children then apply these skills to solving problems involving change in temperature.

Things to look out for

- Children may forget to include zero in a count, for example 3, 2, 1, -1, -2, -3
- Children may not see the reflective nature of negative numbers and count after zero with the negative partner of the first positive number, for example 3, 2, 1, 0, -3, -2, -1

Key questions

- What is a negative number? How do you write negative numbers?
- What is the next number in this count: 3, 2, 1?
- What is the number after that?
- Are the numbers counting forwards or backwards?
- What is the sequence counting forwards/backwards in?
- What number comes before/after _____ when counting forwards/backwards in 1s?

Possible sentence stems

- Numbers less than zero are called _____ numbers.
- I know the numbers are counting forwards/backwards because ...
- The number before/after _____ when counting forwards/backwards in 1s is _____

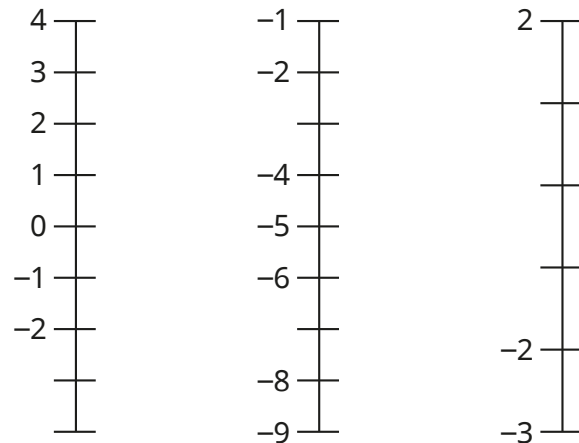
National Curriculum links

- Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

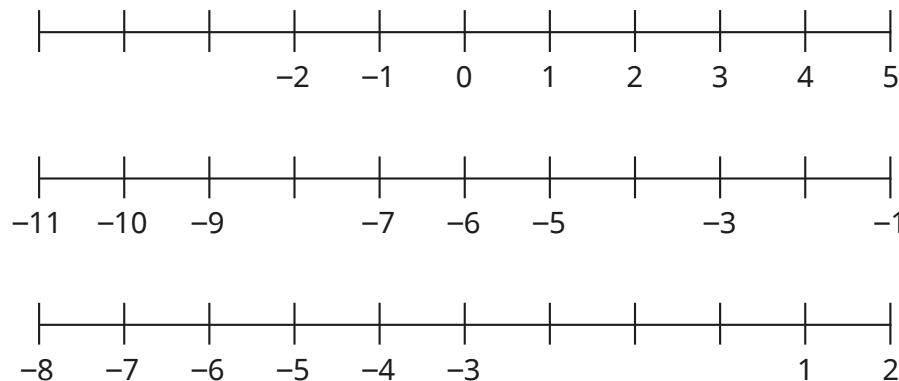
Count through zero in 1s

Key learning

- Work out the missing numbers.



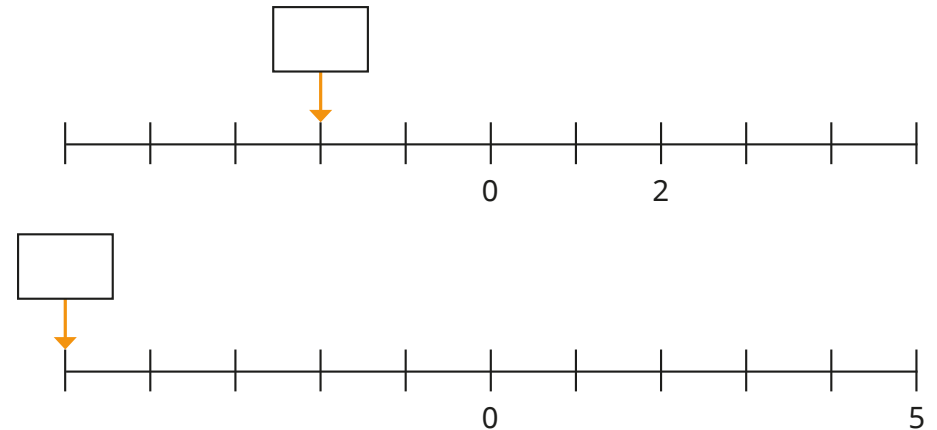
- Complete the number lines.



- What are the next three numbers in each sequence?

- ▶ -20, -19, -18, -17, _____, _____, _____
- ▶ 5, 4, 3, 2, _____, _____, _____
- ▶ -6, -5, -4, -3, _____, _____, _____

- What numbers are the arrows pointing to?



What do you notice?

- The temperature in Halifax is 2 °C.
The temperature in Manchester is 5 degrees colder.
What is the temperature in Manchester?

Count through zero in 1s

Reasoning and problem solving

Ron and Whitney are completing this counting sequence.

4, 3, 2, 1, _____, _____, _____, _____



Ron

The missing numbers are -1, -2, -3, -4

The missing numbers are 0, -4, -3, -2



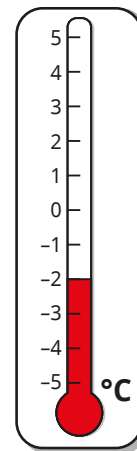
Whitney

What mistake has each child made?

Complete the counting sequence correctly.

0, -1, -2, -3

The thermometer shows the temperature in Helsinki on Monday.



-4 °C

On Tuesday, the temperature was 5 degrees warmer than on Monday.

On Wednesday, the temperature was 7 degrees colder than on Tuesday.

What was the temperature on Wednesday?

Compare methods with a partner.

Count through zero in multiples

Notes and guidance

In this small step, children continue to practise counting both forwards and backwards through zero, but now in multiples other than 1s.

Initially, the focus is on counting where zero is included in the count, which leads to a reflective pattern, for example $-6, -4, -2, 0, 2, 4, 6$. Once children are confident with this, they explore counting through zero that does not follow this pattern, for example $8, 5, 2, -1, -4, -7$. Encourage children to explore how partitioning of the multiple can support counting through zero. For example, when counting back in 5s from 3, they can use the fact that 5 can be partitioned into 3 and 2. This will allow them to first jump to zero and then from zero to reach -2 .

Number lines, both vertical and horizontal, continue to be a key representation in supporting this understanding.

Things to look out for

- In counts that include zero, children may forget to include it.
- Children may just reflect a given sequence rather than counting through zero, for example $-8, -5, -2, 2, 5, 8$
- When counting through zero, children may continue the count from zero, for example $5, 3, 1, 0, -2, -4, -6$

Key questions

- What is the next number in this count: 6, 4, 2?
What is the number after that?
- Are the numbers counting forwards or backwards?
- What is the sequence counting forwards/backwards in?
- What number comes before/after _____ when counting forwards/backwards in _____ s?
- How does partitioning the multiple help when counting through zero?

Possible sentence stems

- The sequence is counting in _____ s.
- The number before/after _____ when counting forwards/backwards in _____ s is _____
- I can partition _____ into _____ and _____ to help count through zero.

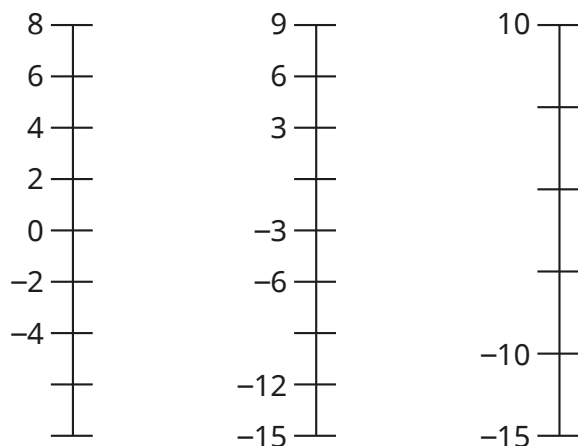
National Curriculum links

- Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

Count through zero in multiples

Key learning

- Work out the missing numbers.



- Complete the sequences.

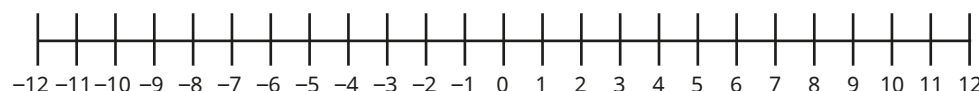
- ▶ -16, -12, -8, ____, ____, ____
- ▶ -5, -10, -15, ____, ____, ____
- ▶ -9, -6, -3, ____, ____, ____
- ▶ 18, 12, 6, ____, ____, ____

- The temperature at 3 pm is 4 °C.

The temperature drops by 2 degrees every hour.

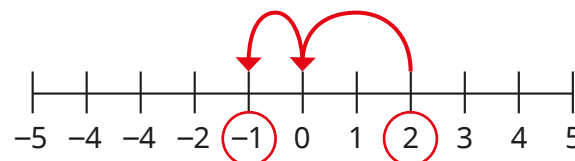
What will the temperature be at 7 pm?

- Use the number line to complete the sequences.

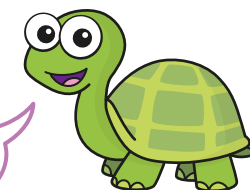


- ▶ 5, 3, 1, ____, ____, ____
- ▶ 7, 4, 1, ____, ____, ____
- ▶ -9, -7, -5, ____, ____, ____
- ▶ -9, -5, -1, ____, ____, ____

- Tiny is counting backwards in 3s from 2



I can partition 3 into 2 and 1 and jump to zero.



Use Tiny's method to find the next number in these counts.

- ▶ counting back in 4s from 2
- ▶ counting back in 5s from 3
- ▶ counting back in 4s from 3
- ▶ counting forwards in 5s from -3

Count through zero in multiples

Reasoning and problem solving

Starting at sea level, a diver descends 5 m every minute for 3 minutes.

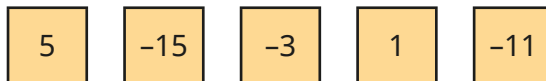
The diver then ascends 3 m every minute until they reach the surface.

How many minutes does it take the diver to reach the surface?



5 minutes

Here are five numbers from a counting pattern.



The numbers are not in the correct order.

A sixth card is missing to complete the counting pattern.

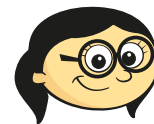
What is the missing number?



-7

Annie and Mo are completing the counting sequence.

8, 5, 2, —, —, —



Annie

The missing numbers are -2, -5, -8

The missing numbers are 0, -3, -6



Mo

-1, -4, -7

What mistake has each child made?

Complete the counting sequence correctly.



Compare and order negative numbers

Notes and guidance

In this small step, children compare and order integers that include negative numbers.

A common misconception is to apply the abstract “rules” of positive numbers to negative numbers. For example, children may believe that because 10 is greater than 3, then -10 must be greater than -3 . Number lines are a key representation to help address this misconception. By comparing positive numbers and reflecting on their positions on a number line, children can begin to generalise that greater numbers lie to the right on a number line. Therefore, because -3 lies to the right of -10 , it is greater. It can also be helpful to discuss real-life contexts to support this understanding. For example, children may be comfortable with the fact that, for example, -5 degrees is colder than -1 degree and can then apply this to show that $-5 < -1$.

Once children are confident with comparing two numbers, they can begin to order groups of integers that include both positive and negative numbers.

Things to look out for

- Directly applying knowledge of comparing and ordering positive numbers can lead children to think that, for example, $-7 > -3$

Key questions

- Where is the number _____ on the number line?
- How can you use a number line to compare numbers?
- When comparing numbers on a number line, are the greater/smaller numbers on the right or the left?
- Are negative numbers greater or less than positive numbers?
- What temperature is warmer/colder, _____ or _____? So which number is greater?
- How do you know that -8 is less than -3 ?

Possible sentence stems

- Greater numbers are to the _____ of smaller numbers on a number line.
- Positive numbers are _____ than negative numbers.
- Ascending/descending order means ordering from _____ to _____

National Curriculum links

- Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

Compare and order negative numbers

Key learning

- Use the number line to help compare the numbers.



$6 \bigcirc 3$

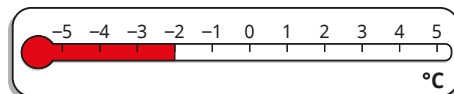
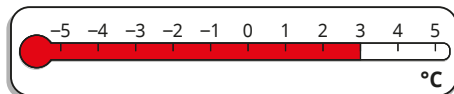
$7 \bigcirc 9$

$2 \bigcirc 0$

Complete the sentence.

Numbers to the left on the number line are _____ than numbers to the right.

- Use the correct word to complete each sentence.



warmer

colder

3 degrees is _____ than -2 degrees.

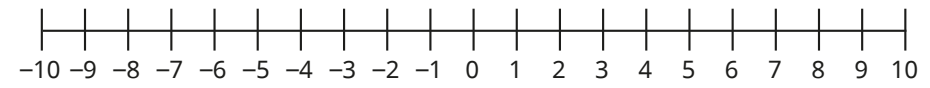


less

greater

-2 degrees is _____ than 3 degrees.

- Use the number line to help compare the numbers.



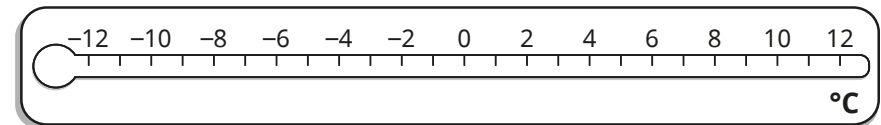
$6 \bigcirc -3$

$-8 \bigcirc -4$

$5 \bigcirc -7$

$0 \bigcirc -5$

- Write the temperatures in order, starting with the coldest.



▶ 9 °C, 0 °C, 3 °C ▶ -9 °C, 0 °C, -3 °C ▶ 8 °C, -1 °C, -3 °C

- Write the numbers in ascending order.

-2 0 7 -7 22 4

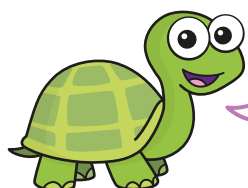
- Write the numbers in descending order.

-41 104 -1 14 -14 4

Compare and order negative numbers

Reasoning and problem solving

Tiny is comparing numbers.



4 is greater than 1, so -4 is greater than -1

Draw a number line and label the positions of the numbers.

Explain why Tiny is incorrect.



Drawing of number line showing that -1 is greater than -4 as it is further to the right

Fill in the missing number.

$$-3 < \boxed{} < 2$$

Find all the possible answers.



$-2, -1, 0, 1$

Amir is on floor 4 of a building.

He gets in a lift and goes down 7 floors.

Rosie is on floor -5 of the building.

She gets in a lift and goes up 3 floors.

Who is on the lower floor now?



Amir

Here are the temperatures in three cities on Monday.

Vancouver	Edinburgh	Stockholm
-7°C	1°C	-3°C

On Tuesday, the temperature in:

- Vancouver is 4 degrees warmer
- Stockholm is 3 degrees warmer
- Edinburgh is 3 degrees colder.

Order the temperatures for Tuesday, starting with the warmest.



Stockholm 0°C
Edinburgh -2°C
Vancouver -3°C

Find the difference

Notes and guidance

In this small step, children look at finding the difference between positive and negative numbers.

As with previous steps, vertical and horizontal number lines are a key representation in supporting this understanding. To begin with, children count either forwards or backwards in 1s through zero, seeing that the difference is the number of jumps between the two numbers. They then look at more efficient strategies by jumping to and from zero and adding the two jumps together to find the difference. For example, to find the difference between -4 and 3 , they can jump 3 from 3 to 0 and then 4 from 0 to -4 . The difference is $3 + 4 = 7$

Contextual problems, such as finding the difference between temperatures or distances above and below ground, are very common, so this step is key for working with negative numbers.

Things to look out for

- When using number lines, children may count the numbers rather than the jumps, resulting in a difference that is 1 greater than it should be.
- Children may rely on always counting individual jumps rather than using the more efficient strategy of jumping to and from zero.

Key questions

- Where is the number _____ on the number line?
- How can you use a number line to find the difference between two numbers?
- How many jumps are there from _____ to _____?
- Does it matter if you count forwards or backwards?
- How far away from zero is _____?
- If the jump from _____ to zero is _____ and the jump from zero to _____ is _____, what is the overall difference?

Possible sentence stems

- There are _____ jumps from _____ to _____, so the difference is _____
- The distance from _____ to zero is _____
The distance from zero to _____ is _____
So the difference between _____ and _____ is _____

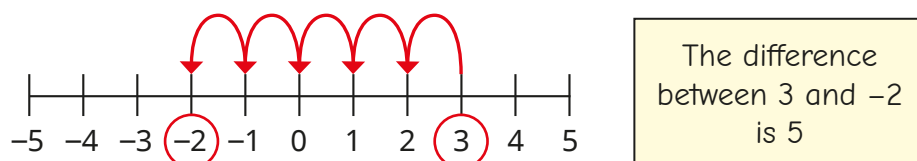
National Curriculum links

- Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

Find the difference

Key learning

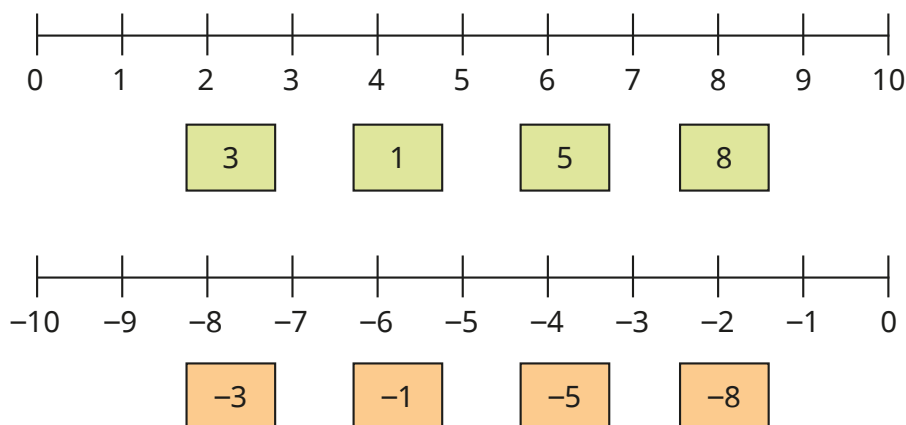
- Max is finding the difference between 3 and -2



Use Max's method to find the differences between the pairs of numbers.

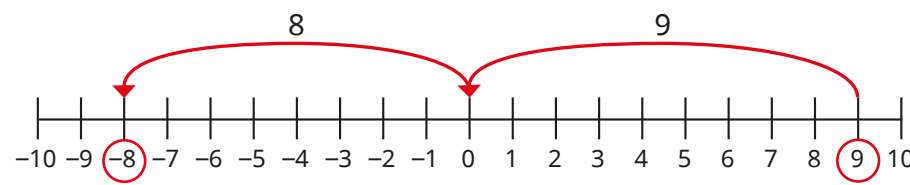
- -1 and 2 ► 2 and -5 ► -2 and 5 ► 3 and -3

- Count the number of jumps from zero to each number.



What do you notice?

- Eva is finding the difference between 9 and -8



$$9 + 8 = 17, \text{ so the difference between 9 and } -8 \text{ is } 17$$

Use Eva's method to find the differences between the pairs of numbers.

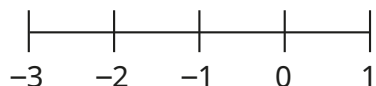
- -5 and 7 ► 8 and -4 ► -1 and 9 ► 6 and -6

- The temperature in London is 8°C .
The temperature in Moscow is -7°C .
How much warmer is the temperature in London than in Moscow?
- Find the differences between the pairs of numbers.
► -32 and 65 ► -48 and 45 ► 132 and -224
- Mrs Fisher parks her car on level -3
Her flat is on level 18
How many floors does she have to go up to get to her flat?

Find the difference

Reasoning and problem solving

Kim is finding the difference between -3 and 1



The difference is 5

What mistake has Kim made?

What is the difference between -3 and 1 ?

4

The table shows the highest and lowest temperatures recorded on a day in two cities.

City	Highest temperature	Lowest temperature
Oslo	4°C	-6°C
Helsinki	3°C	-9°C

Which city has the greater difference in its daily temperature?

Compare methods with a partner.

Helsinki

Jack is finding the difference between -47 and 54

I am going to
count back from 54 to -47
 $54, 53, 52, 51 \dots$



101

Explain a more efficient method
for Jack to find the difference.

What is the difference?

The temperature at 9 am is -5°C .

At 1 pm, the temperature is
9 degrees warmer.

At 9 pm, the temperature has dropped
3 degrees since 1 pm.

What is the difference between the
temperatures at 9 am and 9 pm?

6 degrees

Summer Block 5

Converting units

Small steps

Step 1

Kilograms and kilometres

Step 2

Millimetres and millilitres

Step 3

Convert units of length

Step 4

Convert between metric and imperial units

Step 5

Convert units of time

Step 6

Calculate with timetables

Kilograms and kilometres

Notes and guidance

Children first encountered kilograms in Year 3 and kilometres in Year 4. This small step revisits both of these units of measure and their relationships to grams and metres, respectively.

Begin by discussing what units of measure are and how different units of measure are used for different purposes. Remind children of what kilograms and kilometres are, discussing examples of when each would be used. Then explain that the prefix “kilo-” always means one thousand, so 1,000 grams is equivalent to 1 kilogram and 1,000 metres is equivalent to 1 kilometre. Bar models and double number lines are useful representations for showing the conversions. Make links to multiplying and dividing integers and decimals by 1,000, covered earlier in the year.

Children should also be confident with conversions of simple fractions such as $\frac{1}{2}$ kg = 500 g and $\frac{3}{4}$ km = 750 m.

Things to look out for

- Children may perform the wrong operation, for example multiplying instead of dividing.
- Children may confuse “kilo-” with “centi-” and use the factor of 100 instead of 1,000

Key questions

- What are units of measure?
- What might you measure using kilograms/kilometres?
- What is the same about kilograms and kilometres? What is different?
- What does the prefix “kilo-” mean?
- How many grams are there in _____ kilograms?
- How can you convert from kilometres to metres? What is the same and what is different about converting from metres to kilometres?

Possible sentence stems

- 1 kilometre = _____ m,
so _____ kilometres = _____ \times 1,000 m = _____ m
- _____ g = 1 kg, so _____ g = _____ \div 1,000 = _____ kg

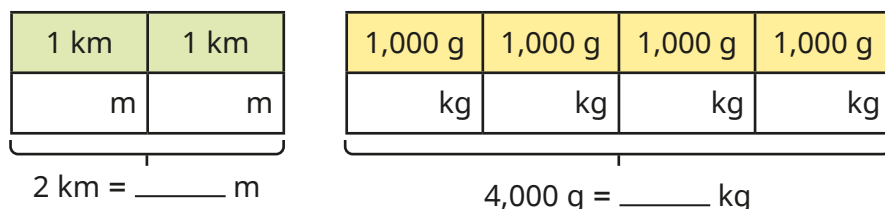
National Curriculum links

- Convert between different units of metric measure [for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre]

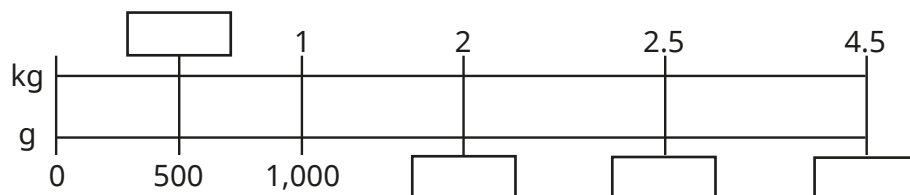
Kilograms and kilometres

Key learning

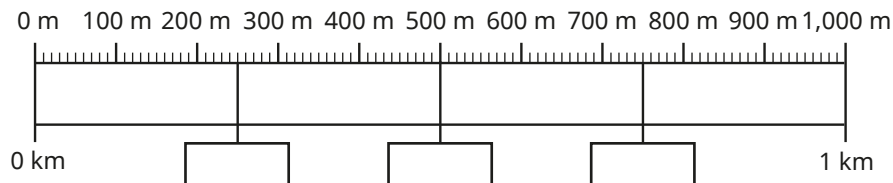
- Complete the bar models.



- Find the missing values on the double number line.



- Use the double number line to help you complete the sentences.



- ▶ 1 km is equivalent to _____ m. ▶ $\frac{1}{4}$ km is equivalent to _____ m.
- ▶ $\frac{1}{2}$ km is equivalent to _____ m. ▶ $\frac{3}{4}$ km is equivalent to _____ m.

- Write <, > or = to compare the measurements.

5 kg ○ 4,500 g 12 kg ○ 12,000 g


3.7 km ○ 370 m 37,000 m ○ 3.7 km

- Fill in the missing numbers.

▶ $\frac{1}{10}$ kg = _____ g ▶ $\frac{3}{10}$ km = _____ m

▶ $7 \text{ kg} + \frac{1}{4} \text{ kg} = \text{_____ g}$ ▶ $12 \text{ km} + \text{_____ km} = 12,500 \text{ m}$

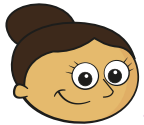
- Eva walks 1,750 m to the bus stop.
She then rides on the bus for 5.2 km.
How far has she travelled in total?

- Each cube has a mass of 250 g. 
How many cubes must be added to balance the scales?



Kilograms and kilometres

Reasoning and problem solving



To convert from kilometres to metres, I multiply by 1,000

$10 \text{ km} = 10 \times 1,000$
 $= 10,000 \text{ m}$ and
 $0 \text{ km} = 0 \times 1,000$
 $= 0 \text{ m}$

Do you agree with Dora?

Explain your answer.

Yes

Dani bakes a cake that has a mass of 2.4 kg.

She cuts it into eight equal pieces.

She eats a piece and gives a piece to each of her two friends.

What is the mass of the remaining cake in grams?

1,500 g

Mr Lee buys 2,500 g of potatoes and 2,000 g of carrots.



78p
per kg



£1.46
per kg

13p

He pays with a £5 note.

How much change does he get?

Mo and Nijah are both doing a sponsored run.

They are each given 25p for every 100 m that they run.

- Mo runs 5.7 km.
- Nijah runs 6,300 m.

£1.50

How much more money does Nijah raise than Mo?

Compare methods with a partner.

Millimetres and millilitres

Notes and guidance

Children first encountered millimetres and millilitres as units of measure in Year 3. In this small step, they convert between millimetres and metres and between millilitres and litres for the first time.

As in the previous step, begin by reminding children what these units of measure are and what they are likely to be used for. Then discuss the prefix “milli-”, explaining that it means one thousandth. Model conversions by multiplying amounts given in litres and metres by 1,000 and dividing amounts given in millimetres and millilitres by 1,000. The use of bar models and double number lines will help children’s understanding of these conversions.

Children then move on to converting amounts given in litres and metres, including decimals and fractions. Finally, they use this understanding to solve problems that require conversions between these units of measure.

Things to look out for

- Children may perform the wrong operation, for example multiplying instead of dividing.
- Children may confuse the different prefixes “kilo-”, “milli-” and “centi-”.

Key questions

- What might you measure in metres/litres?
- What might you measure in millimetres/millilitres?
- What does the prefix “milli-” mean?
- What is the same and what is different about the prefixes “milli-” and “kilo-”?
- How can you convert from litres/metres to millilitres/millimetres?
- How many litres are equivalent to _____ millilitres?
- Which is the greatest length, 1 mm, 1 km or 1 m?
- What unit of measure would you use for measuring _____?

Possible sentence stems

- To convert from litres to millilitres, I _____ by 1,000
- To convert from millimetres to metres, I _____ by 1,000

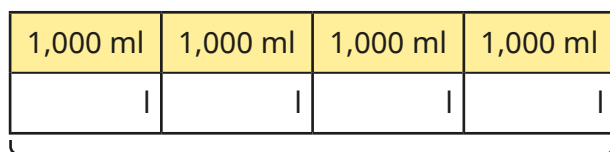
National Curriculum links

- Convert between different units of metric measure [for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre]

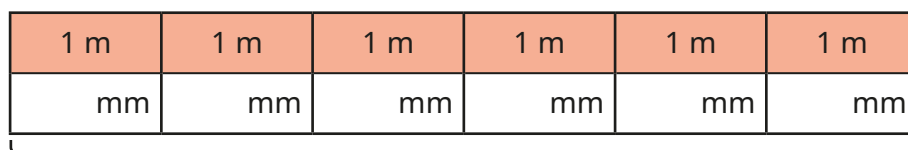
Millimetres and millilitres

Key learning

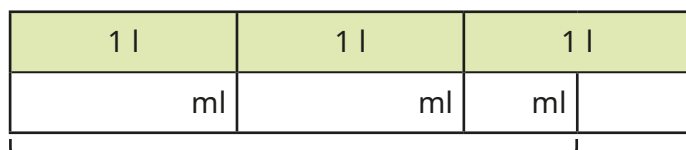
- Use the bar models to complete the conversions.



$$4,000 \text{ ml} = \underline{\hspace{2cm}} \text{ l}$$

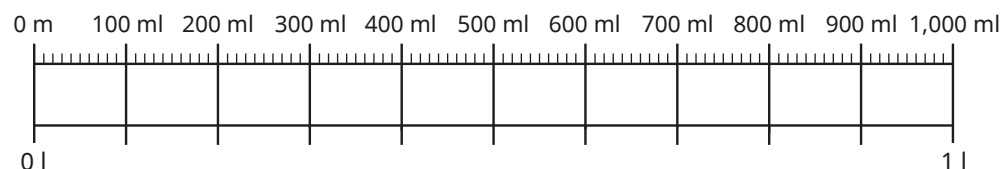


$$6 \text{ m} = \underline{\hspace{2cm}} \text{ mm}$$



$$2 \frac{1}{2} \text{ l} = \underline{\hspace{2cm}} \text{ ml}$$

- Use the double number line to complete the conversions.



- ▶ 1 l = ml ▶ 200 ml = $\frac{\square}{\square}$ l ▶ ml = 0.4 l
- ▶ $\frac{1}{10}$ l = ml ▶ . l = 700 ml

- Use the fact to help you complete the conversions.

$$1,000 \text{ mm} = 1 \text{ m}$$

- ▶ 5,000 mm = m ▶ 500 mm = m
- ▶ 50,000 mm = m ▶ 5,500 mm = m

- Write <, > or = to compare the measurements.

$$2 \text{ l} \bigcirc 1,500 \text{ ml}$$

$$60 \text{ l} \bigcirc 6,000 \text{ ml}$$

$$2.8 \text{ m} \bigcirc 280 \text{ mm}$$

$$3,700 \text{ m} \bigcirc 3.7 \text{ mm}$$

- Fill in the missing numbers.

$$\frac{1}{1000} \text{ m} = \underline{\hspace{2cm}} \text{ mm}$$

$$2 \text{ l} + \underline{\hspace{2cm}} \text{ ml} = 2,500 \text{ ml}$$

$$\frac{1}{100} \text{ m} = \underline{\hspace{2cm}} \text{ mm}$$

$$3 \text{ l} + \frac{1}{4} \text{ l} = \underline{\hspace{2cm}} \text{ ml}$$

$$\frac{1}{10} \text{ m} = \underline{\hspace{2cm}} \text{ mm}$$

$$3 \text{ l} + \underline{\hspace{2cm}} \text{ l} = 3,400 \text{ ml}$$

- Brett has a 2 litre jug of juice.

He pours 350 ml of juice into each of three cups.

How much juice is left in the jug?

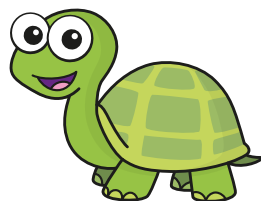
Millimetres and millilitres

Reasoning and problem solving

5 m of ribbon is shared equally between four friends.

5 m			
mm	mm	mm	mm

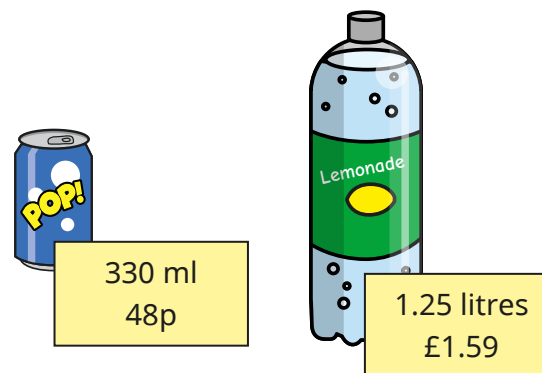
Each friend will receive 125 mm of ribbon.



Do you agree with Tiny?
Explain your answer.

No
There are not
100 mm in 1 m.

Lemonade is sold in cans and bottles.



Alex buys 5 cans and 3 bottles.

She sells the lemonade in 100 ml glasses to raise money for charity.

She sells all the lemonade.

How many glasses does she sell?

Alex charges 50p per glass.

How much profit does she make?

54

£19.83

Convert units of length

Notes and guidance

In this small step, children build on their learning in the previous two steps to convert the units of metric lengths – millimetres, centimetres and metres.

Recap what types of things would be measured by each unit of measure, and when each one would be inappropriate, for example measuring the playground in millimetres or measuring a pencil sharpener in metres. Measuring and drawing lines of specific lengths in centimetres and millimetres help with children's understanding of these measures.

Model how to convert between these units. Begin by discussing the difference between milli- and centi-, meaning that they multiply a length given in metres by 100 to convert it to centimetres, and by 1,000 to convert it to millimetres. Then use division to convert the other way. When children are confident with integer values, they can move on to converting fractional and decimal lengths in metres.

Things to look out for

- Children may confuse when to multiply or divide and/or when to use 10, 100 or 1,000
- Children may confuse the units of measure or omit them from their answers.

Key questions

- What units of length do you know?
- What objects would you measure with millimetres/centimetres/metres?
- Which unit of measure would you use to measure _____?
- How many mm/cm are there in _____ cm/m?
- How can you convert from mm/cm/m to mm/cm/m?
- When do you need to divide/multiply by 10/100/1,000?

Possible sentence stems

- There are _____ mm in _____ cm.
- There are _____ mm in _____ m.
- There are _____ cm in _____ m.
- To convert between mm/cm/m and mm/cm/m, I _____ by _____

National Curriculum links

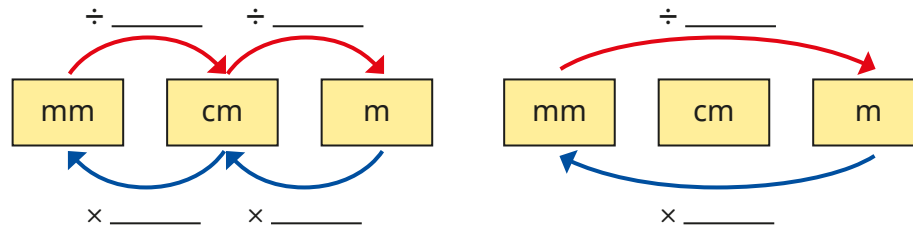
- Convert between different units of metric measure [for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre]

Convert units of length

Key learning

- There are 10 mm in 1 cm and 100 cm in 1 m.

Use this to help you complete the conversion diagrams.



- Fill in the missing numbers in the conversions.

- ▶ 10 mm = _____ cm
- ▶ _____ cm = 1 m
- ▶ 55 mm = _____ cm
- ▶ 300 mm = _____ cm = _____ m
- ▶ _____ mm = 98 cm = _____ m
- ▶ 2 cm = _____ mm
- ▶ _____ m = 300 cm
- ▶ _____ m = 670 cm
- ▶ 5 m = _____ cm
- ▶ 5.6 m = _____ cm

- Measure each line.

Write the lengths in both centimetres and millimetres.



- Here are the heights of four children.

Esther 1.3 m	Scott 124 cm	Aisha 1.32 m	Filip 141 cm
-----------------	-----------------	-----------------	-----------------

Put the children in height order, starting with the shortest.

Write their heights in millimetres.

- Write <, > or = to compare the measurements.

55 mm ○ 6 cm

100 m ○ 1 cm

6.8 cm ○ 7 mm

0.25 m ○ 300 mm

- Line A is 6 centimetres long.

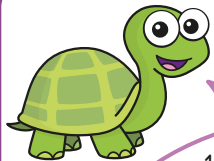
Line B is 54 millimetres longer than line A.

Line C is $\frac{2}{3}$ of line B.

Draw lines A, B and C.

Convert units of length

Reasoning and problem solving



$\frac{1}{2}$ a metre is 500 cm.
That means that $\frac{1}{2}$ a metre
is 5,000 mm because to
convert between cm and mm,
I multiply by 10

Is Tiny correct?

Explain your answer.

No

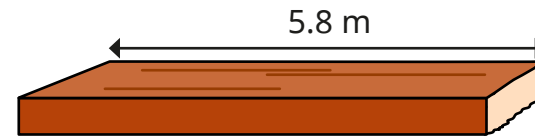
Dexter has a pencil that is
9.5 cm long.

He uses it every day for a week, and it
is now 6.9 cm long.

How many millimetres shorter is his
pencil now than it was a week ago?

26 mm

A plank of wood is 5.8 metres long.



Two lengths are cut from the wood.

175 cm

$3\frac{4}{5}$ m

What length of the plank is left?

25 cm or 0.25 m

A 10p coin is 2 mm thick.

Rosie makes a pile of
10p coins worth £1.30

What is the height
of the pile of coins in
centimetres?



2.6 cm

Convert between metric and imperial units

Notes and guidance

In this small step, children are introduced to imperial units of measure and learn to convert between metric and imperial units.

Begin by having a conversation about different units of measure, asking children to name as many as they can. Sort children's suggestions into metric and imperial units. Explain that the metric and imperial systems are different ways of measuring the same type of thing and it can depend on where you are as to which you use, for example road signs in England are in miles, but in France they are in kilometres.

Model exchanging between the units covered in this step: inches and centimetres, kilograms and pounds, and pints and millilitres. It is important to explain the term “approximately” in this context and that the conversions given are not exact. Explain the meaning of “ \approx ” as “approximately equal to”.

When children are confident converting between units, they can solve problems that include both metric and imperial measures.

Things to look out for

- Children may confuse \approx and $=$.
- Children may forget to include units of measure in their answers.

Key questions

- What different types of units of measure do you know?
- How can you sort the units of measure into groups?
- What is the difference between imperial and metric units of measure?
- What does “approximately equal to” mean? What symbol is used to mean “approximately equal to”?
- How can you convert from cm/kg/ml to inches/lb/pints?
- How can you convert from inches/lb/pints to cm/kg/ml?

Possible sentence stems

- 1 kg is approximately equal to _____ lb, so _____ kg is approximately equal to _____ \times _____ = _____ lb.
- 1 pint is approximately equal to _____ ml, so _____ pints is approximately equal to _____ \times _____ = _____ ml.
- 1 inch is approximately equal to _____ cm, so _____ cm is approximately equal to _____ \div _____ = _____ inches.

National Curriculum links

- Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints

Convert between metric and imperial units

Key learning

- 1 inch is approximately equal to 2.5 cm.

$$1 \text{ inch} \approx 2.5 \text{ cm}$$

Use this fact to complete the conversions.

- 2 inches \approx _____ cm
- 20 inches \approx _____ cm
- _____ inches \approx 7.5 cm
- _____ inches \approx 12.5 cm

- The area of the rectangle is 50 cm²



What is the approximate perimeter of the rectangle in inches?

- 1 kilogram is approximately equal to 2.2 pounds.

$$1 \text{ kg} \approx 2.2 \text{ lb}$$

Use this fact to complete the conversions.

- _____ kg \approx 4.4 lb
- _____ kg \approx 22 lb
- 4 kg \approx _____ lb
- 100 kg \approx _____ lb

- Apples are sold in 2 kg bags.

Huan buys 4 bags of apples.

He uses 2.6 lb of the apples.

What is the approximate mass of Huan's remaining apples in pounds?

- Use the fact to complete the conversions.

$$1 \text{ pint} \approx 568 \text{ ml}$$

- 2 pints \approx _____ ml
- $\frac{1}{2}$ pint \approx _____ ml
- _____ pints \approx 56.8 ml
- _____ pints \approx 5,680 ml

- There are 8 pints in a gallon.

A class is given 2 gallons of lemonade.

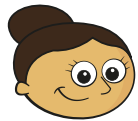
They drink 3 litres of lemonade in total.

About how many millilitres of lemonade do they have left?

1 gallon	1 gallon
pints	pints
ml	ml
_____ ml	

Convert between metric and imperial units

Reasoning and problem solving



Dora

My mass was
7.8 lb when
I was born.

My mass was
3.5 kg when I
was born.



Amir

Who was heavier when they were born,
Dora or Amir?

Explain your answer.

Dora

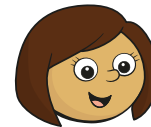


12 cm is greater
than 8 inches because
12 is greater than 8

Is Tiny correct?

Explain your answer.

No



We have 3 pints
of milk delivered to
our house 4 times
a week.



Approximately how many litres of
milk are delivered to Kim's house
each week?

Kim uses about 200 ml of milk every
day on her cereal.

Approximately how many pints of
milk does Kim use for her cereal in
a week?

approximately
6.8 litres

approximately
 $2\frac{1}{2}$ pints

Convert units of time

Notes and guidance

Children have encountered units of time and converted between them in previous years. In this small step, they revisit and extend this learning and solve problems involving units of time.

Ask children to name as many different units for measuring time as they can. Encourage them to think of longer units such as days, weeks, months and years as well as smaller units such as seconds, minutes and hours.

Model the different conversions, many of which, such as days in a week and minutes in an hour, will be familiar from previous learning and everyday experience, but others, such as days in a year or days in different months, may need recapping.

Double number lines are a useful representation to support many of the conversions. Once children are confident converting between different units of time, they can solve problems that involve different units.

Things to look out for

- Children may be confused when converting measures that involve division (for example, days to weeks) if there is a remainder.
- Children may think that time conversions behave like decimals, for example $0.25 \text{ minutes} = 25 \text{ seconds}$.

Key questions

- What units of measure do we use for time?
- How can you put the units of measure for time in order from shortest to longest?
- How many seconds/minutes/hours are there in _____ minutes/hours/days?
- How can you convert from _____ to _____?
- When using division to convert times, what happens if there is a remainder?

Possible sentence stems

- There are _____ seconds/minutes in a minute/hour, so in _____ minutes/hours there are _____ \times _____ = _____ seconds/minutes.
- There are _____ hours in a day, so in _____ hours there are _____ \div _____ = _____ full days and _____ hours.
- To convert _____ into _____, I _____ by _____

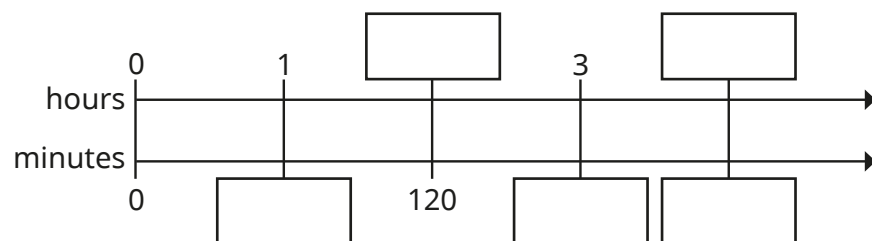
National Curriculum links

- Solve problems involving converting between units of time

Convert units of time

Key learning

- Complete the double number line.



Use the double number line to help work out the conversions.

- ▶ 5 hours = _____ minutes
 - ▶ _____ hours = 600 minutes
 - ▶ $\frac{1}{2}$ hour = _____ minutes
 - ▶ _____ hours = 150 minutes
- There are 60 seconds in a minute.
 - ▶ How many seconds are there in 5 minutes?
 - ▶ How many minutes are equivalent to 630 seconds?
- Sam is boiling an egg.
She wants to boil it for $4\frac{1}{2}$ minutes, but she accidentally boils it for an extra 45 seconds.
How many seconds does she boil the egg for?

- There are 7 days in a full week.
How many full weeks are there in 23 days?
How many days are left over?

- Complete the table.

Days	Weeks and days
42 days	
	5 weeks and 5 days
	10 weeks and 5 days
100 days	

- Complete the conversions.
 - ▶ 1 year = _____ months
 - ▶ _____ years = 60 months
 - ▶ 3 years and 2 months = _____ months
 - ▶ _____ years and _____ months = 75 months
 - ▶ _____ years = 24 months
 - ▶ 2.5 years = _____ months

Convert units of time

Reasoning and problem solving

Whitney, Ron and Tiny are converting units of time.



Whitney

There are
60 seconds in
a minute.

There are
60 minutes
in an hour.



Ron



Tiny

That means that
there are 120 seconds
in an hour.

Do you agree with Tiny?
Explain your answer.

No

Tiny has worked
out $60 + 60$ instead
of 60×60

There are 3,600
seconds in an
hour.

Tom is exactly 11 years old.

There have been two leap years in
his life.

How many days has Tom been alive?

Convert your answer to hours.

Investigate for other ages.



4,017 days
96,408 hours

Three children are running a race.

- Dani finishes the race in 3 minutes and 5 seconds.
- Eva finishes the race in 192 seconds.
- Alex finishes the race in 2 minutes and 82 seconds.

Who wins the race?

Compare methods with a partner.



Dani

Calculate with timetables

Notes and guidance

Earlier in the year, in the statistics block, children read and interpreted timetables. In this small step, this learning is revisited and extended to include using timetables to solve problems that involve calculations with time.

Begin by recapping what timetables are, their purpose and how they are used. Show different timetables and explain how they show what is happening when. Model how to calculate using a timetable, for example lengths of time between events, how long a television programme is, times between stops on a train/bus journey. These can be challenging, especially when the times cross an hour; a number line can be used to support these calculations.

Children answer questions across a range of different timetables, then think of their own questions that could be answered with the information given in a timetable. Finally, children create their own accurate timetable with information provided.

Things to look out for

- Children may confuse 12-hour and 24-hour clock times.
- Children may try to subtract times using the column method, misinterpreting times as decimals.

Key questions

- What information can a timetable give you?
- Why are some parts of the timetable blank?
- How do you convert between times given using 12-hour and 24-hour clocks?
- How long does _____ take?
- How many minutes are there between _____ and _____?
- How can a number line help you to find the difference between two times?
- What questions could you ask about this timetable?

Possible sentence stems

- The _____ train/bus from _____ takes _____ minutes to get to _____
- From _____ to the next hour is _____ minutes.
From _____ to _____ is _____ minutes.
The total time taken is _____ + _____ = _____ minutes.

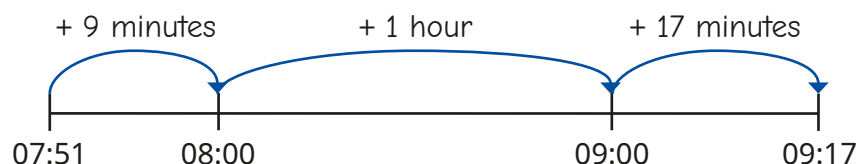
National Curriculum links

- Solve problems involving converting between units of time

Calculate with timetables

Key learning

- Use Mo's number line to work out how long it is between 07:51 and 09:17



- Use the timetable to answer the questions.

Bus Station	06:05	06:35	07:10	07:43	08:15
Shelf Roundabout	06:15	06:45		07:59	08:31
Shelf Village Hall	06:16	06:46	07:25	08:00	08:32
Woodside	06:21	06:50	07:28		
Odsal	06:26	06:55	07:33	08:15	08:45
Railway Station	06:40	07:10	07:48	08:30	09:00

- Why are some of the times blank?
- How long does it take the 06:35 bus to travel from the bus station to Odsal?
- How long does it take the 08:32 bus to get from Shelf Village Hall to the railway station?

- Use the timetable to answer the questions.

	14:01	14:31	15:01	15:31
Ilkley				
Ben Rydding		14:39	15:09	15:39
Burley in Wharfedale	14:12	14:44		15:44
Menston	14:17	14:49	15:15	15:49
Guiselley	14:20		15:18	15:52
Leeds	14:31	14:59	15:29	16:33

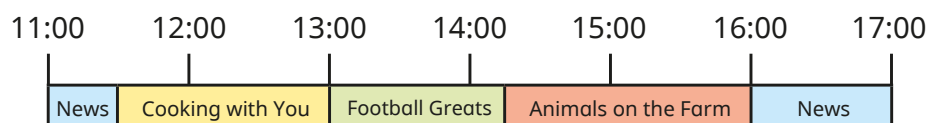
- How long does the 14:01 train from Ilkley take to get to Menston?
- How often do trains leave Ilkley for Leeds?
- How much longer does it take the 15:39 train from Ben Rydding to get to Guiselley than the 15:09 train from Ben Rydding to Guiselley?
- Teddy arrives in Burley in Wharfedale at 2:50 pm. He wants to get to Leeds. When is the earliest he will arrive in Leeds?

Ask a partner more questions that can be answered using the timetable.

Calculate with timetables

Reasoning and problem solving

Here is an extract from a TV guide.



Rosie turns on the TV at 12 noon.

What will be on?

Estimate how long *Animals on the Farm* lasts.

Between 11 am and 5 pm, how many minutes is the news on for altogether?


Ask a partner more questions that can be answered using the guide.

Cooking with You

1 hour and 45 minutes

90 minutes

Here is part of a bus timetable.



Trinity Street	05:40	06:00	06:20	06:35	06:50
Marford Hill	05:51	06:13	06:33	06:48	07:05
Chister Business Park	06:07	06:25	06:48	07:03	07:20
Railway Station	06:18	06:38	07:00	07:15	07:35

Mr Khan is getting the train from the railway station at 07:05

- He lives a 9-minute walk from Marford Hill bus stop.
- The train platform is an 8-minute walk from the railway station bus stop.
- The train journey is 1 hour and 18 minutes.

What time does Mr Khan need to leave his house?

How long will it be from Mr Khan leaving his house to getting off the train?

6:04

2 hours 19 minutes

Summer Block 6

Volume

Small steps

Step 1

Cubic centimetres

Step 2

Compare volume

Step 3

Estimate volume

Step 4

Estimate capacity

Cubic centimetres

Notes and guidance

In Year 3, children compared volumes of liquids using words such as “empty”, “full”, “more” and “less”. In this small step, they learn that volume refers to the amount of three-dimensional space an object takes up, and they measure volume using cubes.

Children make simple shapes with interlocking cubes and describe the volume of each shape in terms of the number of cubes. They then look at pictorial representations and work out how many cubes there are in each shape, including counting the cubes that cannot be seen in the picture. They then find the volume of a variety of shapes, using both concrete and pictorial representations, using the fact that each cube has a volume of one cubic centimetre (written 1 cm^3).

Finally, they make and measure the volumes of cuboids. Children recognise that some of the cubes in a pictorial representation cannot be seen, but that the total volume can be found by counting the number of cubes in each layer. This leads to the formula to work out the volume of a cuboid, which is covered in Year 6

Things to look out for

- Children may only count the visible cubes when working out the volume of a 3-D shape.
- Children may omit units from their answer.

Key questions

- What is volume?
- What unit can you use to measure volume?
- What is the difference between one square centimetre and one cubic centimetre?
- How many cubes is the shape made up of?
- What is the volume of the shape/cuboid?
- How can you make a cuboid that has 16 cubes?

Is there more than one way?

Possible sentence stems

- The number of cubes needed to make the shape is _____
- The volume of the shape is _____ cubic centimetres.
- There are _____ cubes in each layer and there are _____ layers.
There are _____ cubes altogether.

National Curriculum links

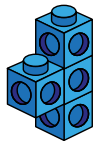
- Estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity

Cubic centimetres

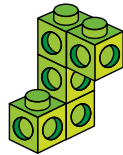
Key learning

- Jack and Kim are using cubes to make shapes.

Jack



Kim

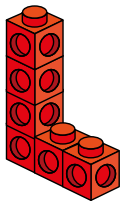


How many cubes have they each used?

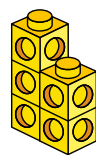
- Dora and Max have each made a shape using cubes.

The volume of each cube is 1 cm^3

Dora



Max

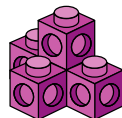


What is the volume of each of their shapes?

- Tommy uses cubes to make this 3-D shape.

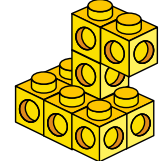
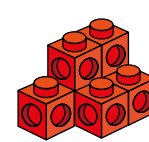
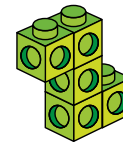
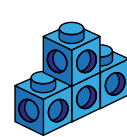
Each cube has a volume of 1 cm^3

What is the volume of Tommy's shape?



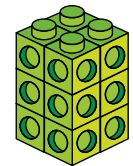
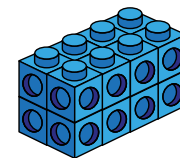
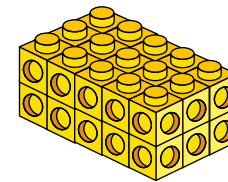
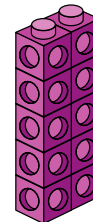
- What is the volume of each 3-D shape?

Each cube has a volume of 1 cm^3



- Rosie makes some cuboids using cubes.

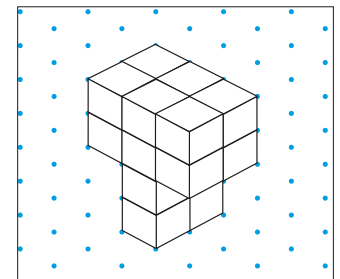
Each cube has a volume of 1 cm^3



What is the volume of each cuboid? How did you work it out?

- Scott draws a "T" shape on isometric paper.

How many cubes does he need to make his 3-D shape?

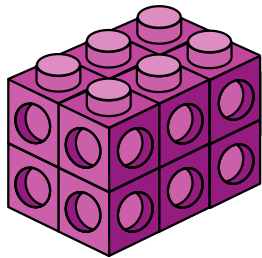


Cubic centimetres

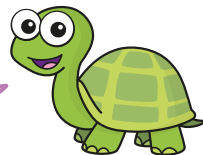
Reasoning and problem solving

Dani makes this cuboid.

Each cube has a volume of 1 cm^3



I can see
10 cubes, which
means that the shape
has a volume
of 10 cm^3



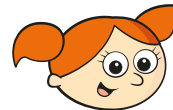
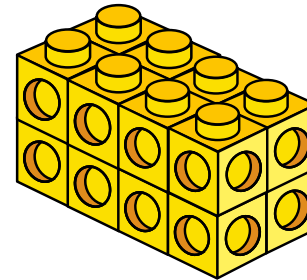
Do you agree with Tiny?
Explain your answer.

No

There are two
cubes that cannot
be seen.

volume = 12 cm^3

Alex is working out the volume of
this cuboid.



I can see
that the top layer is
made up of 8 cubes and
there are 2 layers, so
I can work out the volume
with the multiplication
 8×2

Yes

Is Alex correct?

Explain your answer.



Compare volume

Notes and guidance

This small step builds on the previous step by comparing the volumes of different shapes. In Year 3, children compared the volume of liquid in different containers using simple vocabulary. In this small step, they find the volume of different shapes by counting cubes, then decide which shape has the greater volume.

Begin by looking at 3-D shapes made from interlocking cubes, asking children to say which contains more cubes and so has the greater volume. Children can then move on to pictorial representations, working out the number of cubes needed to make each shape before deciding which has the greater volume.

Finally, children compare cuboids. They may find it easier to make the cuboids themselves in order to work out the volume, or they may count the number of cubes in each layer, then multiply this by the height of the shape.

Things to look out for

- Children may assume that a taller shape always has a greater volume.
- Children may say that a shape with more cubes in it has a greater volume than one with fewer cubes, without considering the sizes of the cubes.

Key questions

- What is volume?
- What is a cubic centimetre?
- How can you find the total volume of the shape?
- What is the volume of shape A?
- How can you tell which shape has the greater volume?
- Which has the greater volume, shape A or shape B?
- Are the cubes the same size? Why does this matter?

Possible sentence stems

- The volume of shape A is _____ and the volume of shape B is _____
Shape _____ has the greater volume.
- To work out the volume of the shape I can...

National Curriculum links

- Estimate volume [for example, using 1 cm³ blocks to build cuboids (including cubes)] and capacity

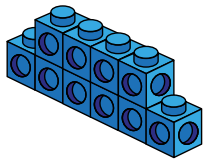
Compare volume

Key learning

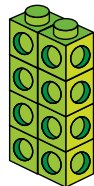
- Dora and Amir each make a shape using cubes.

Each cube has a volume of 1 cm^3

Dora



Amir



My shape
has the greater volume,
because it is taller.

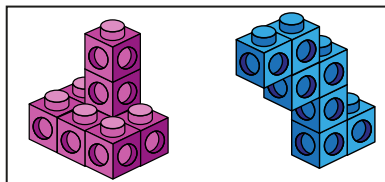
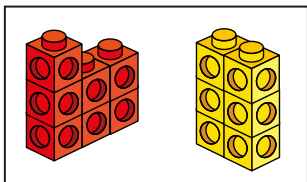
Do you agree with Amir?

Explain your answer.

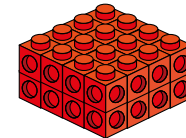
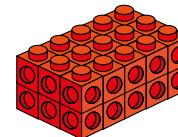
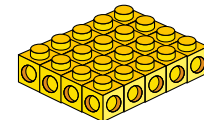
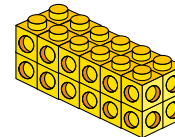
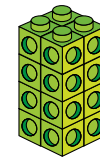
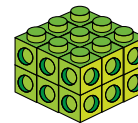
- Each cube has a volume of 1 cm^3

What are the volumes of the shapes?

In each pair, which shape has the greater volume?



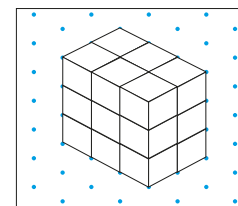
- Write $<$, $>$ or $=$ to compare the volumes of the cuboids.



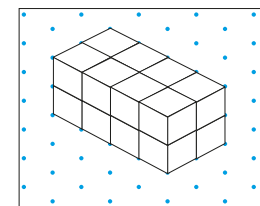
- Dexter and Annie each draw a cuboid on isometric paper.

Whose cuboid has the greater volume?

Dexter



Annie



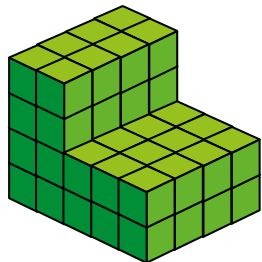
Compare volume

Reasoning and problem solving

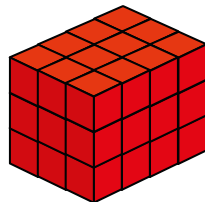
Huan, Esther and Tom each build a shape using cubes.

Each cube has a volume of 1 cm^3

Huan



Esther



Tom's shape has a volume that is greater than Esther's but smaller than Huan's.

What could the volume of Tom's shape be?

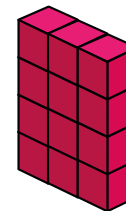
any volume
between 36 cm^3
and 56 cm^3

Jo and Brett each make a shape using cubes.

Jo



Brett



The volume of my shape is 8 cubes and Brett's shape is 12 cubes, so Brett's shape has a greater volume.

Do you agree with Jo?

Explain your answer.

No

Estimate volume

Notes and guidance

In this small step, children estimate the volumes of different objects, by using cubes with a volume of 1 cm^3 and building a shape similar to the 3-D object.

Give children cubes and ask them to estimate the volumes of objects found in the classroom. For example, they could estimate the volume of a small book by making a similar-sized cuboid with interlocking cubes. For each object, discuss whether the actual volume is greater or less than the estimate. For example, an apple may have a smaller volume than that of a similar-sized cuboid.

Children then consider the volumes of much larger objects such as rooms. They discuss why cubic centimetres would be inappropriate for larger volumes and think about the need for different units such as cubic metres.

Things to look out for

- Some objects will be harder to recreate using interlocking cubes than others.
- Children may need support to decide if the estimated volume is greater or less than the actual volume.

Key questions

- What is volume?
- How could you estimate the volume of the shape?
- Which of these two objects has the greater volume?
- How can you use cubes to estimate the volume of an object?
- If object A has a volume of _____, what do you estimate the volume of object B will be?
- Is the actual volume greater or less than the estimated volume?

Possible sentence stems

- I estimate that the volume of _____ is _____ cm^3
- The actual volume of _____ is greater/less than the estimate.

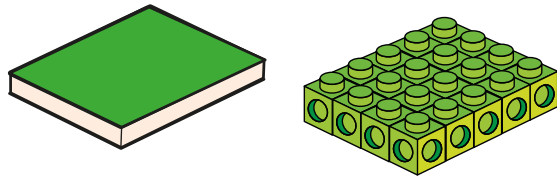
National Curriculum links

- Estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity

Estimate volume

Key learning

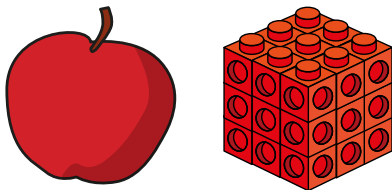
- Mo wants to estimate the volume of the book using cubes. He makes a cuboid.



Work out an estimate for the volume of the book.

Is the actual volume of the book exactly the same as the estimate?

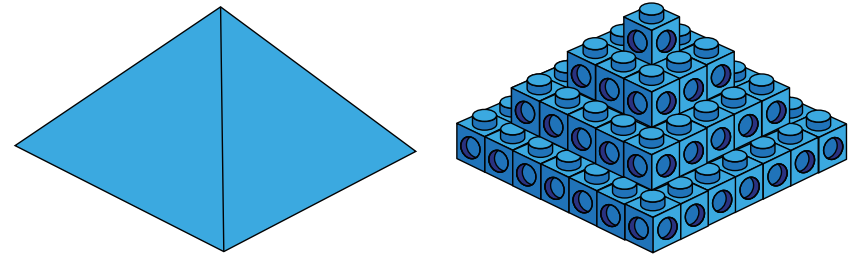
- Aisha is using cubes to estimate the volume of the apple. Each cube has a volume of 1 cm^3



Work out an estimate for the volume of the apple.

Is the actual volume of the apple greater or smaller than the estimate?

- Filip is using cubes to estimate the volume of the pyramid. Each cube has a volume of 1 cm^3



Work out an estimate for the volume of the pyramid.

Is the volume of the pyramid greater or smaller than the estimate?

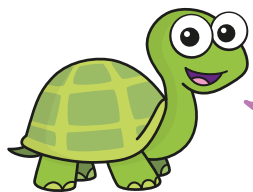
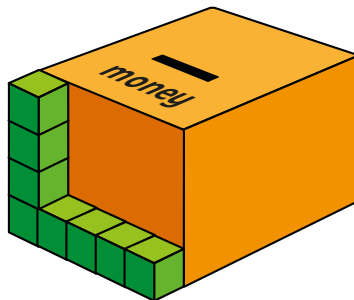
- Why would you not use cubic centimetres to measure the volume of a room?
What different cubic unit could you use instead?
- Estimate the volume of:
 - your classroom
 - the school hall
 - your bedroom

Estimate volume

Reasoning and problem solving

Tiny is using cubes to estimate the volume of a money box.

Each cube has a volume of 1 cm^3



The volume is about 20 cm^3

What mistake has Tiny made?

What is the approximate volume of the money box?

Tiny has not taken into account the depth of the money box.

approximately 100 cm^3

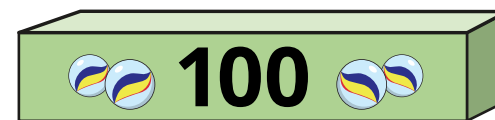
Max has a toy box.



I can fit 8 boxes of marbles in my toy box.

Each box of marbles can hold 100 marbles.

Each marble has a volume of 0.8 cm^3



640 cm^3

Estimate the volume of Max's **toy box**.

Is the actual volume of Max's toy box greater or smaller than your estimate?

Estimate capacity

Notes and guidance

In the final small step of this block, children move on to looking at the capacity of different objects.

Children should be aware of the difference between capacity and volume from earlier learning, knowing that the capacity of, for example, a jug is how much liquid the jug can hold and that volume refers to how much liquid is actually in the jug. They should also know that the term “capacity” is most commonly used when looking at amounts of liquid, and they will have met the measures litres and millilitres as far back as Year 2. They may need reminding that 1 litre is equal to 1,000 millilitres.

Spend some time showing children containers of different sizes, discussing the capacity of each, then matching capacities to containers. Looking at containers that children may be more familiar with, such as a 330 millilitre can and a 2 litre bottle, will help them with estimating the capacity of unknown containers. They can then estimate the capacity of a container where a known amount of something is already inside it.

Things to look out for

- Children may confuse volume and capacity.
- Children may need support to identify which units to use.

Key questions

- What is capacity?
- What is the difference between capacity and volume?
- Which of these containers has the greater capacity? How do you know?
- If there is _____ ml in the jug now, approximately how much will it hold when full?
- What units of measure are used for the capacity of bottles?
- How many millilitres are there in a litre?
- About how many times bigger is the _____ than the _____?

Possible sentence stems

- The capacity of the container is _____ millilitres/litres.
The volume of water in the container is about _____ millilitres/litres.
- Container A is about _____ times the size of container B.

National Curriculum links

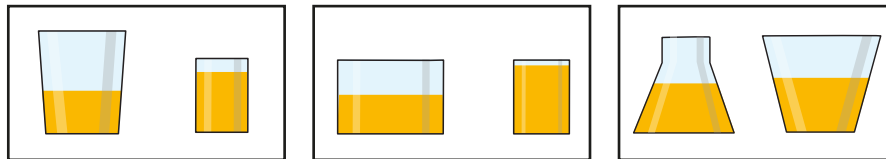
- Estimate volume and capacity [for example, using water]

Estimate capacity

Key learning

- Each pair of containers has the same amount of juice in it.

Which container has the greater capacity in each pair?



- What is the most appropriate capacity of a large bottle of fizzy drink?



20 ml

200 ml

2 litres

20 litres

What is the approximate capacity of a teacup?



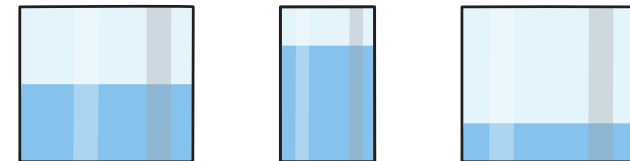
25 ml

150 ml

1.5 litres

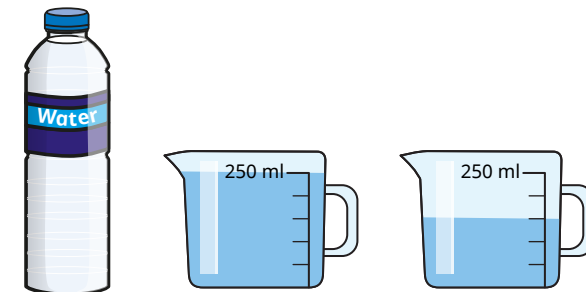
15 litres

- There is 1 litre of water in each container.



Estimate the capacity of each container.

- Sam pours all the water from the bottle into the two containers.



Estimate the capacity of the bottle.

- Each container has a capacity of 1 litre.



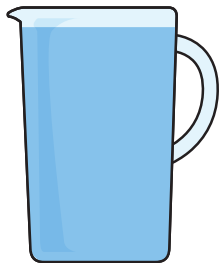
Estimate the volume of water in each container.

Estimate capacity

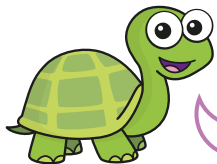
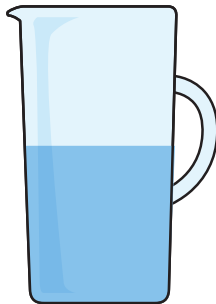
Reasoning and problem solving

There is 500 ml of water in each jug.

A



B



Jug A has a greater capacity than jug B, because the water is higher up the jug.

Do you agree with Tiny?

Explain your answer.

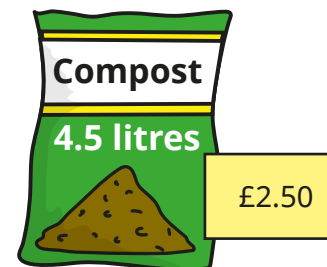
No

1 cubic centimetre of water is the same as 1 millilitre of water and has a mass of 1 gram.

What is the mass of 1 litre of water?

1,000 g or 1 kg

Ron buys compost to fill his flower bed.



He spends £17.50 on compost.

Estimate the capacity of Ron's flower bed.

31.5 litres